*23–10 Lensmaker's Equation

A useful equation, called the **lensmaker's equation**, relates the focal length of a lens to the radii of curvature R_1 and R_2 of its two surfaces and its index of refraction n:

Lensmaker's equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right). \qquad (23\text{–}10)$$

If both surfaces are convex, R_1 and R_2 are considered positive.† For a concave surface, the radius must be considered *negative*.

Notice that Eq. 23–10 is *symmetrical* in R_1 and R_2. Thus, if a lens is turned around so that light impinges on the other surface, the focal length is the same even if the two lens surfaces are different. This confirms what we said earlier: a lens' focal length is the same on both sides of the lens.

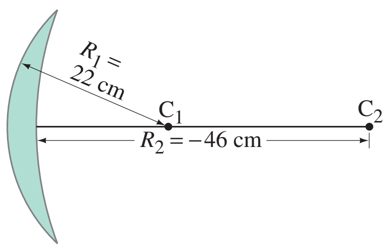

FIGURE 23–46 Example 23–17. The left surface is convex (center bulges outward); the right surface is concave.

EXAMPLE 23–17 Calculating f for a converging lens. A convex meniscus lens (Figs. 23–31a and 23–46) is made from glass with $n = 1.50$. The radius of curvature of the convex surface (left in Fig. 23–46) is 22 cm. The surface on the right is concave with radius of curvature 46 cm. What is the focal length?

APPROACH We use the lensmaker's equation, Eq. 23–10, to find f.

SOLUTION $R_1 = 0.22$ m and $R_2 = -0.46$ m (concave surface). Then

$$\frac{1}{f} = (1.50 - 1.00)\left(\frac{1}{0.22\text{ m}} - \frac{1}{0.46\text{ m}}\right) = 1.19\text{ m}^{-1}.$$

So

$$f = \frac{1}{1.19\text{ m}^{-1}} = 0.84\text{ m},$$

and the lens is converging since $f > 0$.

NOTE If we turn the lens around so that $R_1 = -46$ cm and $R_2 = +22$ cm, we get the same result.

NOTE Because Eq. 23–10 gives $1/f$, it gives directly the power of a lens in diopters, Eq. 23–7. The power of this lens is about 1.2 D.

†Some books use a different convention: R_1 and R_2 may be considered positive if their centers of curvature are to the right of the lens; then a minus sign replaces the + sign in their version of Eq. 23–10.

Summary

Light appears to travel along straight-line paths, called **rays**, through uniform transparent materials including air and glass. When light reflects from a flat surface, the *angle of reflection equals the angle of incidence*. This **law of reflection** explains why mirrors can form **images**.

In a **plane mirror**, the image is virtual, upright, the same size as the object, and as far behind the mirror as the object is in front.

A **spherical mirror** can be concave or convex. A **concave** spherical mirror focuses parallel rays of light (light from a very distant object) to a point called the **focal point**. The distance of this point from the mirror is the **focal length** f of the mirror and

$$f = \frac{r}{2} \qquad (23\text{–}1)$$

where r is the radius of curvature of the mirror.

Parallel rays falling on a **convex mirror** reflect from the mirror as if they diverged from a common point behind the mirror. The distance of this point from the mirror is the focal length and is considered negative for a convex mirror.

For a given object, the approximate position and size of the image formed by a mirror can be found by ray tracing. Algebraically, the relation between image and object distances, d_i and d_o, and the focal length f, is given by the **mirror equation**:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}. \qquad (23\text{–}2)$$

The ratio of image height h_i to object height h_o, which equals the magnification m of a mirror, is

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \qquad (23\text{–}3)$$

If the rays that converge to form an image actually pass through the image, so the image would appear on a screen or film placed there, the image is said to be a **real image**. If the light rays do not actually pass through the image, the image is a **virtual image**.

The speed of light v depends on the **index of refraction**, n, of the material:

$$n = \frac{c}{v}, \qquad (23\text{–}4)$$

where c is the speed of light in vacuum.

When light passes from one transparent medium into another, the rays bend or refract. The **law of refraction** (**Snell's law**) states that

$$n_1 \sin\theta_1 = n_2 \sin\theta_2, \qquad (23\text{–}5)$$

where n_1 and θ_1 are the index of refraction and angle with the normal (perpendicular) to the surface for the incident ray, and n_2 and θ_2 are for the refracted ray.

When light rays reach the boundary of a material where the index of refraction decreases, the rays will be **totally internally reflected** if the incident angle, θ_1, is such that Snell's law would predict $\sin\theta_2 > 1$. This occurs if θ_1 exceeds the critical angle θ_C given by

$$\sin\theta_C = \frac{n_2}{n_1}. \qquad \textbf{(23–6)}$$

A lens uses refraction to produce a real or virtual image. Parallel rays of light are focused to a point, the **focal point**, by a **converging** lens. The distance of the focal point from the lens is the **focal length** f of the lens. It is the same on both sides of the lens.

After parallel rays pass through a **diverging** lens, they appear to diverge from a point in front of the lens, which is its focal point; and the corresponding focal length is considered negative.

The **power** P of a lens, which is $P = 1/f$ (Eq. 23–7), is given in diopters, which are units of inverse meters (m^{-1}).

For a given object, the position and size of the image formed by a lens can be found approximately by ray tracing. Algebraically, the relation between image and object distances, d_i and d_o, and the focal length f, is given by the **thin lens equation**:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}. \qquad \textbf{(23–8)}$$

The ratio of image height to object height, which equals the **magnification** m for a lens, is

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \qquad \textbf{(23–9)}$$

When using the various equations of geometric optics, you must remember the **sign conventions** for all quantities involved: carefully review them (pages 655 and 665) when doing Problems.

[*When two (or more) thin lenses are used in combination to produce an image, the thin lens equation can be used for each lens in sequence. The image produced by the first lens acts as the object for the second lens.]

[*The **lensmaker's equation** relates the radii of curvature of the lens surfaces and the lens' index of refraction to the focal length of the lens.]

Questions

1. Archimedes is said to have burned the whole Roman fleet in the harbor of Syracuse, Italy, by focusing the rays of the Sun with a huge spherical mirror. Is this† reasonable?
2. What is the focal length of a plane mirror? What is the magnification of a plane mirror?
3. Although a plane mirror appears to reverse left and right, it doesn't reverse up and down. Discuss why this happens, noting that front to back is also reversed. Also discuss what happens if, while standing, you look up vertically at a horizontal mirror on the ceiling.
4. An object is placed along the principal axis of a spherical mirror. The magnification of the object is −2.0. Is the image real or virtual, inverted or upright? Is the mirror concave or convex? On which side of the mirror is the image located?
5. If a concave mirror produces a real image, is the image necessarily inverted? Explain.
6. How might you determine the speed of light in a solid, rectangular, transparent object?
7. When you look at the Moon's reflection from a ripply sea, it appears elongated (Fig. 23–47). Explain.

FIGURE 23–47
Question 7.

8. What is the angle of refraction when a light ray is incident perpendicular to the boundary between two transparent materials?
9. When you look down into a swimming pool or a lake, are you likely to overestimate or underestimate its depth? Explain. How does the apparent depth vary with the viewing angle? (Use ray diagrams.)
10. Draw a ray diagram to show why a stick or straw looks bent when part of it is under water (Fig. 23–23).
11. When a wide beam of parallel light enters water at an angle, the beam broadens. Explain.
12. You look into an aquarium and view a fish inside. One ray of light from the fish is shown emerging from the tank in Fig. 23–48. The apparent position of the fish is also shown (dashed ray). In the drawing, indicate the approximate position of the actual fish. Briefly justify your answer.

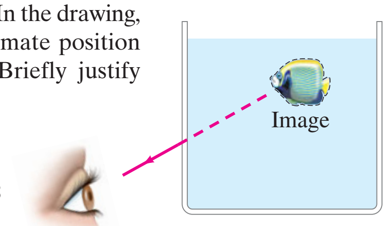

FIGURE 23–48
Question 12.

13. How can you "see" a round drop of water on a table even though the water is transparent and colorless?
14. A ray of light is refracted through three different materials (Fig. 23–49). Which material has (a) the largest index of refraction, (b) the smallest?

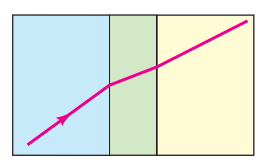

FIGURE 23–49
Question 14.

15. A child looks into a pool to see how deep it is. She then drops a small toy into the pool to help decide how deep the pool is. After this careful investigation, she decides it is safe to jump in—only to discover the water is over her head. What went wrong with her interpretation of her experiment?
16. Can a light ray traveling in air be totally reflected when it strikes a smooth water surface if the incident angle is chosen correctly? Explain.

†Students at MIT did a feasibility study. See www.mit.edu/2.009/www/experiments/deathray/10_ArchimedesResult.html.