*11–15 Mathematical Representation of a Traveling Wave

A simple wave with a single frequency, as in Fig. 11–47, is sinusoidal. To express such a wave mathematically, we assume it has a particular wavelength λ and frequency f. At $t = 0$, the wave shape shown is

$$y = A \sin \frac{2\pi}{\lambda} x, \qquad \textbf{(11–21)}$$

where y is the **displacement** of the wave (either longitudinal or transverse) at position x, λ is the wavelength, and A is the **amplitude** of the wave. [Equation 11–21 works because it repeats itself every wavelength: when $x = \lambda$, $y = \sin 2\pi = \sin 0$.]

Suppose the wave is moving to the right with speed v. After a time t, each part of the wave (indeed, the whole wave "shape") has moved to the right a distance vt. Figure 11–48 shows the wave at $t = 0$ as a solid curve, and at a later time t as a dashed curve. Consider any point on the wave at $t = 0$: say, a crest at some position x. After a time t, that crest will have traveled a distance vt, so its new position is a distance vt greater than its old position. To describe this crest (or other point on the wave shape), the argument of the sine function must have the same numerical value, so we replace x in Eq. 11–21 by $(x - vt)$:

$$y = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right]. \qquad \textbf{(11–22)}$$

Said another way, if you are on a crest, as t increases, x must increase at the same rate so that $(x - vt)$ remains constant.

For a wave traveling along the x axis to the left, toward decreasing values of x, v becomes $-v$, so

$$y = A \sin\left[\frac{2\pi}{\lambda}(x + vt)\right].$$

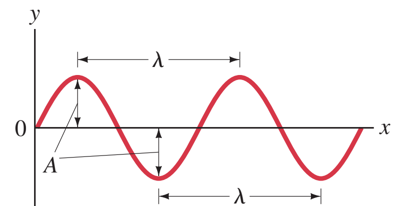
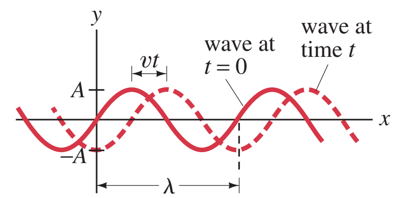


FIGURE 11–47 The characteristics of a single-frequency wave at $t = 0$ (just as in Fig. 11–24).

FIGURE 11–48 A traveling wave. In time t, the wave moves a distance vt.



Summary

An oscillating (or vibrating) object undergoes **simple harmonic motion** (SHM) if the restoring force is proportional to (the negative of) the displacement,

$$F = -kx. \qquad \textbf{(11–1)}$$

The maximum displacement from equilibrium is called the **amplitude**.

The **period**, T, is the time required for one complete cycle (back and forth), and the **frequency**, f, is the number of cycles per second; they are related by

$$f = \frac{1}{T}. \qquad \textbf{(11–2)}$$

The period of oscillation for a mass m on the end of a spring is given by

$$T = 2\pi\sqrt{\frac{m}{k}}. \qquad \textbf{(11–6a)}$$

SHM is **sinusoidal**, which means that the displacement as a function of time follows a sine curve.

During SHM, the total energy

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \qquad \textbf{(11–3)}$$

is continually changing from potential to kinetic and back again.

A **simple pendulum** of length ℓ approximates SHM if its amplitude is small and friction can be ignored. For small amplitudes, its period is given by

$$T = 2\pi\sqrt{\frac{\ell}{g}}, \qquad \textbf{(11–11a)}$$

where g is the acceleration of gravity.

When friction is present (for all real springs and pendulums), the motion is said to be **damped**. The maximum displacement decreases in time, and the mechanical energy is eventually all transformed to thermal energy.

If a varying force of frequency f is applied to a system capable of oscillating, the amplitude of oscillation can be very large if the frequency of the applied force is near the **natural** (or **resonant**) **frequency** of the oscillator. This is called **resonance**.

Vibrating objects act as sources of **waves** that travel outward from the source. Waves on water and on a cord are examples. The wave may be a **pulse** (a single crest), or it may be continuous (many crests and troughs).

The **wavelength** of a continuous sinusoidal wave is the distance between two successive crests.

The **frequency** is the number of full wavelengths (or crests) that pass a given point per unit time.

The **amplitude** of a wave is the maximum height of a crest, or depth of a trough, relative to the normal (or equilibrium) level.

The **wave speed** (how fast a crest moves) is equal to the product of wavelength and frequency,

$$v = \lambda f. \qquad \textbf{(11–12)}$$

In a **transverse wave**, the oscillations are perpendicular to the direction in which the wave travels. An example is a wave on a cord.

In a **longitudinal wave**, the oscillations are along (parallel to) the line of travel; sound is an example.

Waves carry energy from place to place without matter being carried. The **intensity** of a wave is the energy per unit time carried across unit area (in watts/m^2). For three-dimensional waves traveling outward from a point source, the intensity decreases inversely as the square of the distance from the source (ignoring damping):

$$I \propto \frac{1}{r^2}. \qquad \textbf{(11–16b)}$$

Wave intensity is proportional to the amplitude squared and to the frequency squared.

Waves reflect off objects in their path. When the **wave front** (of a two- or three-dimensional wave) strikes an object, the **angle of reflection** is equal to the **angle of incidence**. This is the **law of reflection**. When a wave strikes a boundary between two materials in which it can travel, part of the wave is reflected and part is transmitted.

When two waves pass through the same region of space at the same time, they **interfere**. The resultant displacement at any point and time is the sum of their separate displacements (= the **superposition principle**). This can result in **constructive interference**, **destructive interference**, or something in between, depending on the amplitudes and relative phases of the waves.

Waves traveling on a string of fixed length interfere with waves that have reflected off the end and are traveling back in the opposite direction. At certain frequencies, **standing waves** can be produced in which the waves seem to be standing still rather than traveling. The string (or other medium) is vibrating as a whole. This is a resonance phenomenon, and the frequencies at which standing waves occur are called **resonant frequencies**. Points of destructive interference (no oscillation) are called **nodes**. Points of constructive interference (maximum amplitude of vibration) are called **antinodes**.

[*Waves change direction, or **refract**, when traveling from one medium into a second medium where their speed is different. Waves spread, or **diffract**, as they travel and encounter obstacles. A rough guide to the amount of diffraction is $\theta \approx \lambda/\ell$, where λ is the wavelength and ℓ the width of an obstacle or opening. There is a significant "shadow region" only if the wavelength λ is smaller than the size of the obstacle.]

[*A traveling wave can be represented mathematically as $y = A \sin\{(2\pi/\lambda)(x \pm vt)\}$.]

Questions

1. Is the acceleration of a simple harmonic oscillator ever zero? If so, where?
2. Real springs have mass. Will the true period and frequency be larger or smaller than given by the equations for a mass oscillating on the end of an idealized massless spring? Explain.
3. How could you double the maximum speed of a simple harmonic oscillator (SHO)?
4. If a pendulum clock is accurate at sea level, will it gain or lose time when taken to high altitude? Why?
5. A tire swing hanging from a branch reaches nearly to the ground (Fig. 11–49). How could you estimate the height of the branch using only a stopwatch?

FIGURE 11–49 Question 5.

6. For a simple harmonic oscillator, when (if ever) are the displacement and velocity vectors in the same direction? When are the displacement and acceleration vectors in the same direction?
7. Two equal masses are attached to separate identical springs next to one another. One mass is pulled so its spring stretches 40 cm and the other is pulled so its spring stretches only 20 cm. The masses are released simultaneously. Which mass reaches the equilibrium point first?
8. What is the approximate period of your walking step?
9. What happens to the period of a playground swing if you rise up from sitting to a standing position?
10. Why can you make water slosh back and forth in a pan only if you shake the pan at a certain frequency?
11. Is the frequency of a simple periodic wave equal to the frequency of its source? Why or why not?
12. Explain the difference between the speed of a transverse wave traveling along a cord and the speed of a tiny piece of the cord.
13. What kind of waves do you think will travel along a horizontal metal rod if you strike its end (*a*) vertically from above and (*b*) horizontally parallel to its length?
14. Since the density of air decreases with an increase in temperature, but the bulk modulus B is nearly independent of temperature, how would you expect the speed of sound waves in air to vary with temperature?
15. If a rope has a free end, a pulse sent down the rope behaves differently on reflection than if the rope has that end fixed in position. What is this difference, and why does it occur?
16. How did geophysicists determine that part of the Earth's interior is liquid?