Summary

When a rigid object rotates about a fixed axis, each point of the object moves in a circular path. Lines drawn perpendicularly from the rotation axis to various points in the object all sweep out the same angle θ in any given time interval.

Angles are conventionally measured in **radians**, where one radian is the angle subtended by an arc whose length is equal to the radius, or

$$2\pi \text{ rad} = 360^\circ$$
$$1 \text{ rad} \approx 57.3^\circ.$$

Angular velocity, ω, is defined as the rate of change of angular position:

$$\omega = \frac{\Delta\theta}{\Delta t}. \quad \textbf{(8–2)}$$

All parts of a rigid object rotating about a fixed axis have the same angular velocity at any instant.

Angular acceleration, α, is defined as the rate of change of angular velocity:

$$\alpha = \frac{\Delta\omega}{\Delta t}. \quad \textbf{(8–3)}$$

The linear velocity v and acceleration a of a point located a distance r from the axis of rotation are related to ω and α by

$$v = r\omega, \quad \textbf{(8–4)}$$
$$a_{\tan} = r\alpha, \quad \textbf{(8–5)}$$
$$a_{\mathrm{R}} = \omega^2 r, \quad \textbf{(8–6)}$$

where $a_{\tan}$ and a_{R} are the tangential and radial (centripetal) components of the linear acceleration, respectively.

The frequency f is related to ω by

$$\omega = 2\pi f, \quad \textbf{(8–7)}$$

and to the period T by

$$T = 1/f. \quad \textbf{(8–8)}$$

If a rigid object undergoes uniformly accelerated rotational motion (α = constant), equations analogous to those for linear motion are valid:

$$\omega = \omega_0 + \alpha t, \qquad \theta = \omega_0 t + \tfrac{1}{2}\alpha t^2,$$
$$\omega^2 = \omega_0^2 + 2\alpha\theta, \qquad \bar{\omega} = \frac{\omega + \omega_0}{2}. \quad \textbf{(8–9)}$$

The **torque** due to a force $\vec{\mathbf{F}}$ exerted on a rigid object is equal to

$$\tau = r_\perp F = rF_\perp = rF\sin\theta, \quad \textbf{(8–10)}$$

where $r_\perp$, called the **lever arm**, is the perpendicular distance from the axis of rotation to the line along which the force acts, and θ is the angle between $\vec{\mathbf{F}}$ and r.

The rotational equivalent of Newton's second law is

$$\Sigma\tau = I\alpha, \quad \textbf{(8–14)}$$

where $I = \Sigma mr^2$ is the **moment of inertia** of the object about the axis of rotation. I depends not only on the mass of the object but also on how the mass is distributed relative to the axis of rotation. For a uniform solid cylinder or sphere of radius R and mass M, I has the form $I = \frac{1}{2}MR^2$ or $\frac{2}{5}MR^2$, respectively (see Fig. 8–20).

The **rotational kinetic energy** of an object rotating about a fixed axis with angular velocity ω is

$$\mathrm{KE} = \tfrac{1}{2}I\omega^2. \quad \textbf{(8–15)}$$

For an object both translating and rotating, the total kinetic energy is the sum of the translational kinetic energy of the object's center of mass plus the rotational kinetic energy of the object about its center of mass:

$$\mathrm{KE} = \tfrac{1}{2}Mv_{\mathrm{CM}}^2 + \tfrac{1}{2}I_{\mathrm{CM}}\,\omega^2 \quad \textbf{(8–16)}$$

as long as the rotation axis is fixed in direction.

The **angular momentum** L of an object rotating about a fixed rotation axis is given by

$$L = I\omega. \quad \textbf{(8–18)}$$

Newton's second law, in terms of angular momentum, is

$$\Sigma\tau = \frac{\Delta L}{\Delta t}. \quad \textbf{(8–19)}$$

If the net torque on an object is zero, $\Delta L/\Delta t = 0$, so L = constant. This is the **law of conservation of angular momentum** for a rotating object.

The following Table summarizes angular (or rotational) quantities, comparing them to their translational analogs.

Translation	Rotation	Connection
x	θ	$x = r\theta$
v	ω	$v = r\omega$
a	α	$a_{\tan} = r\alpha$
m	I	$I = \Sigma mr^2$
F	τ	$\tau = rF\sin\theta$
$\mathrm{KE} = \frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	
$p = mv$	$L = I\omega$	
$W = Fd$	$W = \tau\theta$	
$\Sigma F = ma$	$\Sigma\tau = I\alpha$	
$\Sigma F = \frac{\Delta p}{\Delta t}$	$\Sigma\tau = \frac{\Delta L}{\Delta t}$	

[*Angular velocity, angular acceleration, and angular momentum are vectors. For a rigid object rotating about a fixed axis, the vectors $\vec{\boldsymbol{\omega}}$, $\vec{\boldsymbol{\alpha}}$, and $\vec{\mathbf{L}}$ point along the rotation axis. The direction of $\vec{\boldsymbol{\omega}}$ or $\vec{\mathbf{L}}$ is given by the **right-hand rule**.]