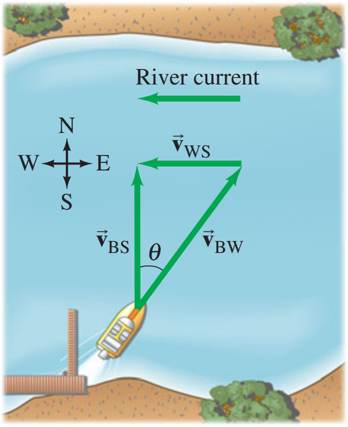


FIGURE 3–29 Example 3–10.

EXAMPLE 3–10 **Heading upstream.** A boat's speed in still water is $v_{BW} = 1.85\ \text{m/s}$. If the boat is to travel north directly across a river whose westward current has speed $v_{WS} = 1.20\ \text{m/s}$, at what upstream angle must the boat head? (See Fig. 3–29.)

APPROACH If the boat heads straight across the river, the current will drag the boat downstream (westward). To overcome the river's current, the boat must have an upstream (eastward) component of velocity as well as a cross-stream (northward) component. Figure 3–29 has been drawn with $\vec{\mathbf{v}}_{BS}$, the velocity of the **B**oat relative to the **S**hore, pointing directly across the river because this is where the boat is supposed to go. (Note that $\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}$.)

SOLUTION Vector $\vec{\mathbf{v}}_{BW}$ points upstream at angle θ as shown. From the diagram,

$$\sin\theta = \frac{v_{WS}}{v_{BW}} = \frac{1.20\ \text{m/s}}{1.85\ \text{m/s}} = 0.6486.$$

Thus $\theta = 40.4°$, so the boat must head upstream at a 40.4° angle.

EXAMPLE 3–11 **Heading across the river.** The same boat $(v_{BW} = 1.85\ \text{m/s})$ now heads directly across the river whose current is still $1.20\ \text{m/s}$. (*a*) What is the velocity (magnitude and direction) of the boat relative to the shore? (*b*) If the river is 110 m wide, how long will it take to cross and how far downstream will the boat be then?

APPROACH The boat now heads directly across the river and is pulled downstream by the current, as shown in Fig. 3–30. The boat's velocity with respect to the shore, $\vec{\mathbf{v}}_{BS}$, is the sum of its velocity with respect to the water, $\vec{\mathbf{v}}_{BW}$, plus the velocity of the water with respect to the shore, $\vec{\mathbf{v}}_{WS}$: just as before,

$$\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}.$$

SOLUTION (*a*) Since $\vec{\mathbf{v}}_{BW}$ is perpendicular to $\vec{\mathbf{v}}_{WS}$, we can get v_{BS} using the theorem of Pythagoras:

$$v_{BS} = \sqrt{v_{BW}^2 + v_{WS}^2} = \sqrt{(1.85\ \text{m/s})^2 + (1.20\ \text{m/s})^2} = 2.21\ \text{m/s}.$$

We can obtain the angle (note how θ is defined in Fig. 3–30) from:

$$\tan\theta = v_{WS}/v_{BW} = (1.20\ \text{m/s})/(1.85\ \text{m/s}) = 0.6486.$$

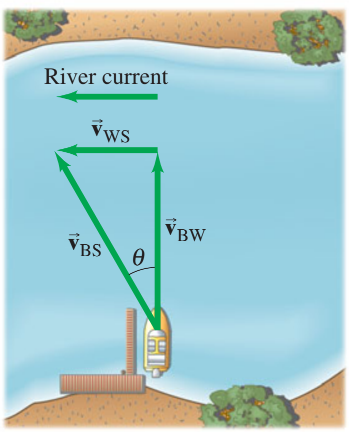
A calculator with a key INV TAN or ARC TAN or TAN^{-1} gives $\theta = \tan^{-1}(0.6486) = 33.0°$. Note that this angle is not equal to the angle calculated in Example 3–10.

(*b*) The travel time for the boat is determined by the time it takes to cross the river. Given the river's width $D = 110\ \text{m}$, we can use the velocity component in the direction of D, $v_{BW} = D/t$. Solving for t, we get $t = 110\ \text{m}/1.85\ \text{m/s} = 59.5\ \text{s}$. The boat will have been carried downstream, in this time, a distance

$$d = v_{WS}t = (1.20\ \text{m/s})(59.5\ \text{s}) = 71.4\ \text{m} \approx 71\ \text{m}.$$

NOTE There is no acceleration in this Example, so the motion involves only constant velocities (of the boat or of the river).

FIGURE 3–30 Example 3–11. A boat heading directly across a river whose current moves at 1.20 m/s.



Summary

A quantity such as velocity, that has both a magnitude and a direction, is called a **vector**. A quantity such as mass, that has only a magnitude, is called a **scalar**. On diagrams, vectors are represented by arrows.

Addition of vectors can be done graphically by placing the tail of each successive arrow at the tip of the previous one. The sum, or **resultant vector**, is the arrow drawn from the tail of the first vector to the tip of the last vector. Two vectors can also be added using the parallelogram method.

Vectors can be added more accurately by adding their **components** along chosen axes with the aid of trigonometric functions. A vector of magnitude V making an angle θ with the $+x$ axis has components

$$V_x = V\cos\theta, \qquad V_y = V\sin\theta. \qquad \textbf{(3–3)}$$

Given the components, we can find a vector's magnitude and direction from

$$V = \sqrt{V_x^2 + V_y^2}, \qquad \tan\theta = \frac{V_y}{V_x}. \qquad \textbf{(3–4)}$$

Projectile motion is the motion of an object in the air near the Earth's surface under the effect of gravity alone. It can be analyzed as two separate motions if air resistance can be ignored. The horizontal component of motion is at constant velocity, whereas the vertical component is at constant acceleration, $\vec{\mathbf{g}}$, just as for an object falling vertically under the action of gravity.

The velocity of an object relative to one frame of reference can be found by vector addition if its velocity relative to a second frame of reference, and the **relative velocity** of the two reference frames, are known.