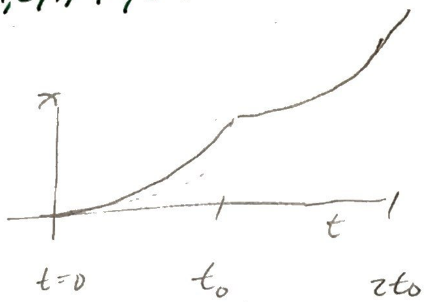
BRO. Evenson PHYS 321 ROY W. ERICKSON

Chpt 2: Newtonian Mechanics

Ass # 2 1, 2, 3, 4, 5, 6, 7

2 1

8/10



$t = 0, t_0$ $F = F_0$ mass = m

$t = t_0, 2t_0$ $F = 2F_0$

Find the total displacement and speed @ $t = 2t_0$

$t = (0, t_0)$

$$m\ddot{x} = F_0 \quad \text{or} \quad m\frac{dv}{dt} = F_0 \qquad \int_{v=0}^{v_0} m\,dv = \int F_0\,dt$$

$$\Rightarrow \quad mv\Big]_0^{v} = F_0 t\Big]_0^{t_0} \quad \text{or} \quad \boxed{mv = F_0 t} \quad \underline{@\ t_0} \rightarrow \boxed{v_0 = \frac{F_0 t_0}{m}}$$

$$v = \frac{F_0 t}{m} \quad \text{or} \quad \frac{dx}{dt} = \frac{F_0 t}{m} \qquad x\Big]_{x_0=0}^{x} = \frac{F_0 t^2}{2m}\Big]_{t=0}^{t_0} \qquad \boxed{x = \frac{F_0 t_0^2}{2m} \ @\ t_0}$$

$t = (t_0, 2t_0)$

$$m\ddot{x} = 2F_0 \qquad \int_{v_0}^{v} dv = \frac{2F_0}{m}\int_{t_0}^{t} dt \quad \text{or} \quad (v - v_0) = \frac{2F_0}{m}(t - t_0)$$

$$\text{so} \quad v(t) = \frac{2F_0}{m}(t - t_0) + v_0$$

$$\underline{v(2t_0)} = \frac{2F_0}{m}(2t_0 - t_0) + \left(\frac{F_0 t_0}{m}\right)$$

$$= \frac{2F_0 t_0}{m} + \frac{F_0 t_0}{m} = \boxed{\frac{3F_0 t_0}{m}}$$

$$v = \frac{2F_0}{m}(t - t_0) + v_0$$

$$x\Big]_{x_0}^{x} = \int_{t_0}^{t}\left[\frac{2F_0}{m}(t - t_0) + v_0\right]dt = \frac{2F_0}{2m}t^2 - \frac{2F_0 t_0 t}{m} + v_0 t\Big]_{t_0}^{t}$$

$$(x - x_0) = \frac{F_0 t^2}{m} - \frac{2F_0 t_0 t}{m} + v_0 t - \frac{F_0 t_0^2}{m} + \frac{2F_0 t_0 t_0}{m} - v_0 t_0$$

$t = 2t_0$
$t_0 = t_0$

$$x(2t_0) = \frac{F_0(2t_0)^2}{m} - \frac{2F_0 t_0(2t_0)}{m} + v_0(2t_0) - \frac{F_0 t_0^2}{m} + \frac{2F_0 t_0^2}{m} - v_0 t_0$$

(2.1 cont.)

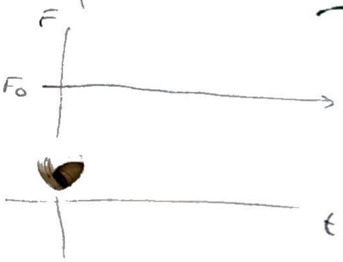
$$x(2t_0) = \frac{4F_0t_0^2}{m} - \frac{4F_0t_0^2}{m} + 2t_0\left(\frac{F_0t_0}{m}\right) - \frac{F_0t_0^2}{m} + \frac{2F_0t_0^2}{m} - \left(\frac{F_0t_0}{m}\right)t_0$$

$$= \frac{4F_0t_0^2}{m} - \frac{4F_0t_0^2}{m} + \frac{2F_0t_0^2}{m} - \frac{F_0t_0^2}{m} + \frac{2F_0t_0^2}{m} - \frac{F_0t_0^2}{m}$$

$$= \frac{8F_0t_0^2}{m} - \frac{6F_0t_0^2}{m} = \boxed{\frac{2F_0t_0^2}{m} = x(2t_0)}$$ X −2

2. $\boxed{2}$ Given the following forces on a particle
Find $v(t)$ and $x(t)$ for particle m where $t_0 = 0$, $v_0 = 0$

10/10 for (a) $F = F_0$

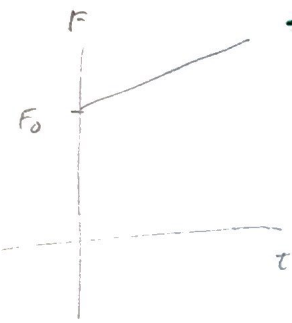


$$m\ddot{x} = F_0 = m\dot{v} \qquad \int_{v_0=0}^{v} dv = \int_{t_0=0}^{t} \frac{F_0}{m}dt$$

$$V = F_0 t \text{ or } \boxed{v(t) = \frac{F_0 t}{m}}$$

$$\int_{x_0}^{x} dx = \int_{t_0=0}^{t} \frac{F_0 t}{m}dt \rightarrow x - x_0 = \frac{F_0t^2}{2} \qquad \boxed{x(t) = \frac{F_0t^2}{2m} + \cancel{x_0}}\ 0$$

(b) $F = F_0 + bt$



$$\int_0^v dv = \int_0^t \left(\frac{F_0 + bt}{m}\right)dt = \frac{F_0}{m}t + \frac{bt^2}{2m}\Big]_0^t$$

or

$$\boxed{v(t) = \frac{F_0 t}{m} + \frac{bt^2}{2m}}$$

$$\int_{x_0}^{x} dx = \int_{t=0}^{t}\left(\frac{F_0 t}{m} + \frac{bt^2}{2m}\right)dt \qquad x - x_0 = \frac{F_0t^2}{2m} + \frac{bt^3}{6m}\Big]_0^t$$

$$\boxed{x(t) = \frac{F_0t^2}{2m} + \frac{bt^3}{6m} + \cancel{x_0}}\ 0$$

(2.2 cont.)

$u = \omega t$, $du = \omega\, dt$

(c) $F = F_0 \cos\omega t$

$$\int_0^v dv = \int_0^t \frac{F_0 \cos\omega t}{m}\, dt \qquad v = \frac{F_0 \sin\omega t}{m\omega}$$

$$\boxed{v(t) = \frac{F_0 \sin\omega t}{\omega m}}$$

$$v(t) = \frac{dx(t)}{dt} \quad \text{so} \quad \int_{x_0}^{x} dx = \int_{t=0}^{t} \frac{F_0 \sin\omega t}{\omega m} \qquad (x - x_0) = -\frac{F_0 \cos\omega t}{\omega^2 m}\Big|_0^t$$

$$\boxed{x(t) = -\frac{F_0 \cos\omega t}{\omega^2 m} + x_0}$$

$$x_0 = \frac{F_0}{m\omega^2}$$

$$\frac{F_0}{\omega^2 m}(1 - \cos\omega t)$$

(d) $F = kt^2$

$$\int_0^v dv = \int_0^t \frac{kt^2}{m}\, dt \qquad \boxed{\frac{kt^3}{3m} = v(t)}$$

$$\int_{x_0}^{x} dx = \int_0^t \frac{kt^3}{3m}\, dt \quad \text{or} \quad \boxed{x(t) = \frac{kt^4}{12m} + \cancel{x_0}}$$

at $t = 0$, $x_0 = 0$

2 3. For a particle under the following forces
Find $V(x)$ for m if $v_0=0,\ x=0$

10 (a) $F = F_0 + kx$

$$m\ddot{x} = m\frac{d\dot{x}}{dt} = \frac{dv}{dt}\frac{dx}{dx} = v\frac{dv}{dx}m$$

so $$F = \boxed{m\ddot{x} = vm\frac{dv}{dx} = F_0 + kx}$$

or $$\int_{v=0}^{v} v\,dv = \int_{x=0}^{x}\left(\frac{F_0 + kx}{m}\right)dx$$

$$\frac{v^2}{2} = \frac{F_0 x}{m} + \frac{kx^2}{2m}$$

so $$\boxed{V(x) = \pm\sqrt{\frac{2F_0 x}{m} + \frac{kx^2}{m}}}$$ ✓

(b) $F = F_0 e^{-kx}$

$$\int_0^v v\,dv = \int_0^x \frac{F_0}{m} e^{-kx}\,dx = \frac{(V(x))^2}{2} = \frac{F_0}{m}\frac{e^{-kx}}{-k}\Big|_0^x$$ or

$$\boxed{V(x) = \pm\sqrt{\frac{2F_0}{-km}}\, e^{\frac{-kx}{2}}}$$ ✗ −2

$$\frac{v^2}{2} = \frac{F_0}{-km}\left[e^{-kx} - 1\right]$$

$$\boxed{V = \pm\sqrt{\frac{2F_0}{km}\left[1 - e^{-kx}\right]}}$$

(c) $F = F_0 + kv$

$$mv\frac{dv}{dx} = F_0 + kv \quad \text{or} \quad \int_0^v \frac{mv\,dv}{(F_0 + kv)} = \int_0^x dx$$

$u = F_0 + kv \quad du = k\,dv$

$$x = m\int_0^v \left(\frac{1}{k} - \frac{F_0/k}{F_0 + kv}\right)dv = \frac{m}{k}\int_0^v dv - \frac{F_0 m}{k}\int_0^v \frac{dv}{F_0 + kv}$$

$$x = \frac{mv}{k} - \frac{F_0 m}{k^2}\ln|F_0 + kv|\Big|_0^v$$

$$\boxed{x = \frac{mv}{k} - \frac{F_0 m}{k^2}\ln\left|\frac{F_0 + kv}{F_0}\right|}$$ −2

initial conditions or boundary conditions must be satisfied

2 $\boxed{4}$ For a particle of mass m under the force ...

$$F(x) = -kx^n$$

(a) Find the potential energy funct.
(b) if $v = v_0$ @ $t=0$ & @ $x=0$ find $v(x)$.
(c) determine the turning points of the motion.

(a)

$$F = -\frac{\partial V}{\partial r} = -\frac{dV}{dx}$$

10

$$\int dV = \int -F\,dx = \int +kx^n dx$$

$$\text{so } V = \frac{kx^{n+1}}{n+1}\Big]_0^{x_0} + c$$

$$\boxed{V(x) = \frac{kx^{n+1}}{n+1}}$$

(b) find $v(x)$

$$v\frac{dv}{dx}m = -kx^n \qquad \frac{v^2}{2}m\Big]_{v_0}^{v} = -k\int_0^x x^n dx$$

$$\frac{mv^2}{2} - \frac{mv_0^2}{2} = -\frac{kx^{n+1}}{n+1}$$

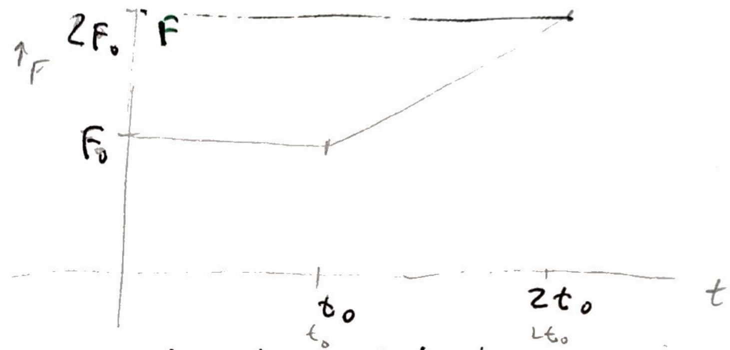
$$v^2 = -\frac{k2x^{n+1}}{m(m+1)} + v_0^2 \quad \text{or} \quad \boxed{V(x) = \pm\sqrt{\frac{-2kx^{n+1}}{m(n+1)} + v_0^2}}$$

(c)

the quantity in the radical must be ≥ 0 for real velocities, (or say $V=0$ at turning points)

$$\text{so } v_0^2 = \frac{2kx^{n+1}}{m(n+1)} \quad \text{or} \quad \boxed{\left(\frac{m(n+1)v_0^2}{2k}\right)^{\frac{1}{n+1}} = x}$$

2 $\boxed{5}$



$v_0 = 0$

A particle is subjected to the force shown above.

show total distance $2t_0 \Rightarrow \frac{13}{6}\frac{F_0 t_0^2}{m}$

$t = (0, t_0)$

$m\ddot{x} = m\dot{v} = F_0 \qquad \int_0^v dv = \frac{F_0}{m}\int_0^{t_0} dt \qquad \underline{v = \frac{F_0 t}{m}} \qquad v(t_0) = \frac{F_0 t_0}{m}$ ✓

$\int_{x_0}^{x} dx = \frac{F_0}{m}\int_{t_0}^{t} t\,dt \qquad (x - x_0) = \frac{F_0}{m}\frac{t^2}{2}\Big]_{t_0}^{t} \qquad \underline{\frac{F_0}{2m}(t^2 - \overset{0}{\cancel{t_0^2}}) + x_0 = x}$

$t = (t_0, 2t_0)$

(t_0, t_0)

$F(t) = \frac{F_0}{t_0} t$ ✓ $t > t_0$ $\qquad \int_{v_0}^{v} dv = \frac{F_0}{m t_0}\int_{t_0} t\,dt \qquad v - v_0 = \frac{F_0}{2t_0}(t^2 - t_0^2)$

$\boxed{v = \frac{F_0}{m 2t_0}(t^2 - t_0^2) + v_0}$

$\int_{x_0'}^{x} dx = \frac{F_0}{2mt_0}\int_{t_0}^{t}(t^2 - t_0^2)\,dt + \int_{t_0}^{t} v_0\,dt$

$\boxed{(x - x_0')} = \frac{F_0}{2mt_0}\left[\frac{t^3}{3} - t_0^2 t\right]_{t_0}^{t} + v_0 t\Big]_{t_0}^{t} = \boxed{\frac{F_0}{2mt_0}\left[\frac{t^3}{3} - t_0^2 t - \frac{t_0^3}{3} + t_0^2 t_0\right] + v_0(t - t_0)}$

for $t = 2t_0$

$(x - x_0') = \frac{F_0}{2mt_0}\left[\frac{(2t_0)^3}{3} - t_0^2(2t_0) - \frac{t_0^3}{3} + t_0^2(t_0)\right] + v_0(2t_0 - t_0)$

$\boxed{x} = \frac{F_0}{2mt_0}\left[\frac{8t_0^3}{3} - 2t_0^3 - \frac{t_0^3}{3} + t_0^3\right] + v_0 t_0 + x_0'$ ✓

$= \frac{F_0 t_0^3}{2mt_0}\left[\frac{8}{3} - 2 - \frac{1}{3} + 1\right] + \left(\frac{F_0 t_0}{m}\right)t_0 + \frac{F_0}{2m}(2t_0)^2 = \frac{F_0 t_0^2}{2m}\left[\left(\frac{4}{3}\right) + 1 + 2\right] = \boxed{\frac{F_0 t_0^2}{2m}\left(\frac{13}{3}\right)}$

$8/3 - \frac{1}{3} - \frac{6}{3} + \frac{3}{3}$

2 $\boxed{6}$

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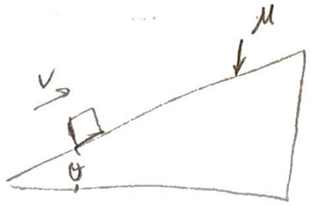
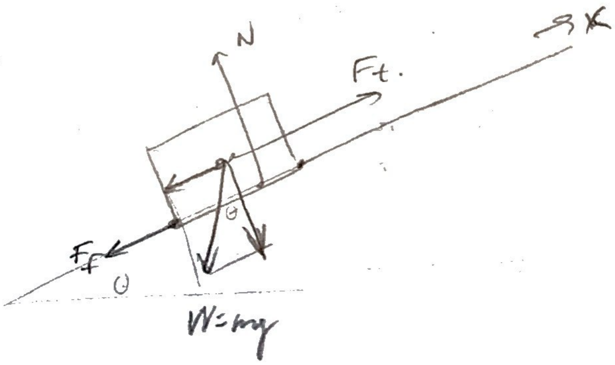


$v_0 = v_0$ @ $t=0$

• Find total time for block to return to starting point.

• For what μ will it just come to rest as it returns to initial point



$$\qquad W_x \qquad + \quad F_f$$

$$F_t = \cancel{m}g\sin\theta + \mu \cancel{m}g\cos\theta = \cancel{m}\ddot{x}$$

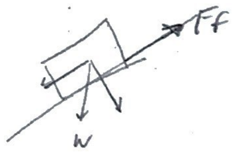
$$g(\sin\theta + \mu\cos\theta) = \ddot{x}$$

$$\int_{v_0}^{v} dv = \int_0^t g(\sin\theta+\mu\cos\theta)\,dt$$

$$v = g(\sin\theta+\mu\cos\theta)t + v_0$$

So the time up from $x=0$ to resting place is

when $v(t)=0$ $\quad -g(\sin\theta+\mu\cos\theta)t = v_0$ $\quad \boxed{t_{up} = \left|\frac{-v_0}{g(\sin\theta+\mu\cos\theta)}\right|}$



$$\int v\,dt = \int g(s\theta+\mu c\theta)t + v_0$$

$$x = \frac{gt^2}{2}(s\theta+\mu c\theta)+v_0 t = x_{up}$$

$$-m\ddot{x} + \mu gm\cos\theta - gm\sin\theta = 0$$

$$-\int dv = \int g(s\theta - \mu c\theta)\,dt \rightarrow v = g(-s\theta-\mu c\theta)t$$

or $x = \frac{gt^2}{2}(\mu c\theta - s\theta)$

$x_{up} = x_{down}$ or $\frac{gt_{up}^2}{2}\overbrace{(s\theta+\mu c\theta)}^{+} + v_0 t_{up} = \frac{gt_d^2}{2}\overbrace{(\mu c\theta - s\theta)}^{-}$

$$\frac{gt_u^2(+)}{2} + v_0 t_u = \frac{gt_d^2}{2}(-)$$

$$\frac{g}{2}\left(\frac{v_0}{g(+)}\right)^2(+) + V_0 t_u = \frac{g}{2}t_d^2(-)$$

(2.6 cont)

$$\frac{\cancel{g}\, v_0^2 \cancel{(+)}}{2g^{\cancel{2}}(+)^{\cancel{2}}} + v_0 \frac{v_0}{g(+)} = \frac{g}{2} t_d^2 (-)$$

$$\frac{v_0^2}{g(+)}\left[\underbrace{\frac{1}{2}+1}_{3/2}\right]\frac{2}{g(-)} = t_d^2$$

$$\frac{3v_0^2}{g^2(+)(-)} = t_d^2 \quad \text{or} \quad \frac{\sqrt{3}\, v_0}{g}\frac{1}{\sqrt{(+)(-)}} = t_d$$

So time up + time down = total

$$\frac{v_0 \sqrt{3}}{g\sqrt{(s\theta + \mu c\theta)(\mu c\theta - s\theta)}} + \frac{v_0}{g(s\theta + \mu c\theta)} = t_{TOTAL}$$

$$\boxed{\frac{v_0}{g}\left(\frac{\sqrt{3}}{\sqrt{\mu^2 c^2\theta - s^2\theta}} + \frac{1}{(s\theta + \mu c\theta)}\right) = t_T}$$

-1

For $v_d = 0$ at initial point

$v_d = g(-s\theta - \mu c\theta)\, t$

$v_d(t_d) = 0 = g(\underbrace{-s\theta - \mu c\theta}_{=0})\, t_d$

so $s\theta = -(\cos\theta)\mu$

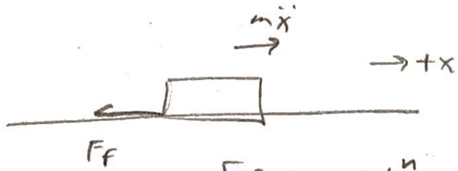
$\boxed{\tan(-\theta) = \mu}$ ×

$$t = \frac{v_0}{g}\left[\frac{1}{\sin\theta + \mu\cos\theta} + \frac{1}{\sqrt{\sin^2\theta - \mu^2\cos^2\theta}}\right]$$

-1

No value of μ will cause the block to come to rest as it returns to the initial point because the acceleration is constant.

2 [7]



$v = v_0$ @ $t=0$ find ① $v(t)$, ② $x(t)$, ③ $v(x)$

$F_f = -cv^n$

④ show for $n=\frac{1}{2}$ $x_T = \frac{2nv_0^{3/2}}{3c}$

equation of motion

$$m\ddot{x} = -cv^n$$

$$m\,dv = -cv^n\,dt$$

$$\int_{v_0}^{v}\frac{dv}{v^n} = \frac{-c}{m}\int_0^t dt \qquad n \neq 1$$

$$\left.\frac{v^{-n+1}}{-n+1}\right]_{v_0}^{v} = \frac{-c}{m}t \qquad \text{or} \qquad \frac{(v^{1-n} - v_0^{1-n})}{1-n} = \frac{-c}{m}t$$

so
$$\boxed{v(t) = \left[\left(\frac{-ct}{m}\right)(1-n) + v_0^{1-n}\right]^{\frac{1}{1-n}}} \checkmark \quad \text{correct.} \quad n \neq 1$$

$u = ct - b \quad du = c\,dt$

$$\int_{x = x_0}^{x} dx = \int_{t_0=0}^{t} v(t)\,dt = \int_{t_0=0}^{t}(Ct+b)^{\frac{1}{1-n}}\,dt = \left.\frac{(Ct+b)^{1+\frac{1}{1-n}}}{\left(1+\frac{1}{1-n}\right)C}\right]_{t_0=0}^{t}$$

$C = \frac{-c(1-n)}{m}$

$b = v_0^{1-n}$

so

$$x - x_0^{\,=0} = \frac{\left(\left(\frac{-ct}{m}\right)(1-n) + v_0^{1-n}\right)^{1+\frac{1}{1-n}} - \left(v_0^{1-n}\right)^{1+\frac{1}{1-n}}}{\left(1+\frac{1}{1-n}\right)\left(\frac{(1-n)(-c)}{m}\right)} = \frac{\left[\frac{c(n-1)}{m}t + v_0^{1-n}\right]^{1+\frac{1}{1-n}} - v_0^{2-n+\frac{1}{1-n}}}{\frac{(n-2)c}{m}}$$

$$\left(1+\frac{1}{1-n}\right)\left(\frac{(1-n)(-c)}{m}\right) = \frac{(n-2)c}{m}$$

$$\boxed{x(t) = \frac{\left[\frac{c(n-1)}{m}t + v_0^{1-n}\right]^{\frac{n-2}{n-1}} - \left(v_0^{1-n}\right)^{\frac{n-2}{n-1}}}{\frac{c(n-2)}{m}} + \cancel{x_0}} \checkmark \text{ o.k.}$$

(book's way) ✗

can be simplified to
$$x(t) = \frac{m}{c(2-n)}\left[v_0^{2-n} - \left(v_0^{1-n} - \frac{c(1-n)}{m}t\right)^{\frac{2-n}{1-n}}\right] \qquad n \neq 1, 2$$

$v(t) = v_0 e^{-(c/m)t}$, $n=1$; $\quad x = \frac{mv_0}{c}\left(1 - e^{-\frac{ct}{m}}\right)$, $n=1$

— 1

(2.7 cont)

$$m v \frac{dv}{dx} = -c v^n \quad \text{or} \quad \int_{v_0}^{v} \frac{v\,dv}{v^n} = \int_{x_0}^{x} -\frac{c}{m}\,dx$$

$$\int_{v_0}^{v} v^{1-n}\,dv = -\frac{c}{m} x \Big]_{x_0}^{x}$$

$$\frac{v^{1-n+1}}{1-n+1}\Big]_{v_0}^{v} = -\frac{c}{m}(x-x_0) \quad \text{or} \quad \frac{v^{2-n} - v_0^{2-n}}{2-n} = -\frac{c}{m}(x-x_0)$$

$n \neq 2$

so

$$v^{2-n} = -\frac{c}{m}(2-n)(x-x_1) + v_0^{2-n}$$

$$\boxed{v(x) = \left[\frac{c}{m}(n-2)(x-x_0) + v_0^{2-n}\right]^{\frac{1}{2-n}}}$$

$v(x) = 0$ when its stops so $\frac{c}{m}(n-2)(x-0) + v_0^{2-n} = 0$

$$\frac{c}{m}\left(-\frac{3}{2}\right)x + v_0^{2-n} = 0 \qquad \boxed{x = \frac{2 m v_0^{3/2}}{3c}}$$

✓

Assig 3: $\begin{cases} 2.9, 11, 12, 15, 18, 23 \\ 3.1, 2, 3, 4 \end{cases}$ PHYS 321 ROY W. Erickson

$\boxed{\text{General 3-D motion}}$

$\boxed{2.9}$ Find the relationship between x and v for (a) $F \propto v$ (b) $F \propto v^2$ starting from rest.

(a)

eq. of motion, $\boxed{m\ddot{x} + cv + mg = 0}$ x is positive upward.

$\uparrow F_f$ $\downarrow F_g$ $\downarrow F_T$

$$m\ddot{x} = mv\frac{dv}{dx} \qquad mv\frac{dv}{dx} + (cv + mg) = 0$$

$$\boxed{\frac{m\,v}{(cv+mg)}dv + dx = 0} \qquad m\int_{v=0}^{v}\frac{v\,dv}{cv+mg} + \int_0^x dx = 0$$

$$\frac{v}{cv+mg} = \frac{1}{c} + \frac{-mg/c}{cv+mg} \qquad m\int_0^v\left[\frac{1}{c} + \frac{-mg/c}{cv+mg}\right]dv + x = 0$$

$u = cv + mg$
$du = c\,dv$

$$\frac{m}{c}\int_0^v dv + \frac{-m^2 g}{c^2}\int_0^v \frac{du}{u} + x = 0 \qquad \frac{mv}{c} - \frac{m^2 g}{c^2}\ln|cv+mg|\Big]_0^v + x = 0$$

or

$$\frac{mv}{c} - \frac{m^2 g}{c^2}\ln|cv+mg| + \frac{m^2 g}{c^2}\ln|mg| + x = 0$$

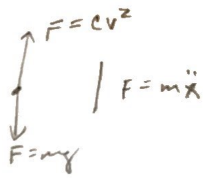
so

$$x(v) = -\frac{mv}{c} + \frac{m^2 g}{c^2}\ln\left|\frac{cv+mg}{mg}\right| = \boxed{-\frac{m}{c}v + \frac{m^2 g}{c^2}\ln\left|\frac{c}{mg}v + 1\right| = x(v)}$$

distance fallen is $-x$

so

$$\boxed{d = \frac{mv}{c} - \frac{m^2 g}{c^2}\ln\left|1 + \frac{cv}{mg}\right|}$$



(2.9)

(b) $F = +cv^2$ again '+' is upward.

$$\boxed{m\ddot{x} - cv^2 + mg = 0}$$

$$m v\frac{dv}{dx} - (cv^2+mg) = 0 \quad \text{or} \quad \int_0^v \frac{m v\,dv}{(-cv^2+mg)} + \int_0^x dx = 0$$

$$\frac{mv}{-cv^2+mg} \qquad \frac{\frac{m}{c}v}{-v^2+\frac{mg}{c}} \ : \text{let } u = -v^2+\frac{mg}{c} \qquad du = -2v\,dv$$

$$\int_0^v \frac{-\left(\frac{m}{c}\right)v\,\frac{du}{2v}}{u} = \frac{-m}{2c}\ln\left|-v^2+\frac{mg}{c}\right|\Bigg]_0^v = -\frac{m}{2c}\left[\ln\left|-v^2+\frac{mg}{c}\right| - \ln\left|\frac{mg}{c}\right|\right]$$

So

$$x - \frac{m}{2c}\ln\left|\frac{-v^2+\frac{mg}{c}}{mg/c}\right| = 0 \Rightarrow \boxed{x(v) = \frac{m}{2c}\ln\left|\frac{-cv^2}{mg}+1\right|}$$

distance fallen is $-x$

so

$$\boxed{d = -\frac{m}{2c}\ln\left|1-\frac{cv^2}{mg}\right|}$$

Given | Find

2.11 $v(x) = \frac{b}{x}$ | $F(x) = ?$

5/5

$F = m\ddot{x} = m v \frac{dv}{dx}$ $\quad \frac{dv(x)}{dx} = \frac{-b}{x^2}$

$dx^{-1} \quad -1x$

$F(x) = m\left(\frac{b}{x}\right)\left(\frac{-b}{x^2}\right) = \boxed{\frac{-mb^2}{x^3} = F(x)}$

2.12 $F(x,v) = f(x)g(v)$ show solvable by sep. of vars

5/10

$m\ddot{x} = F(x)g(v)$ $\quad m\ddot{x} \equiv mv\frac{dv}{dx} = f(x)g(v)$

by separation of variables,

$$\frac{mv\,dv}{g(v)} = f(x)\,dx$$

or by integrating $\boxed{m\int \frac{v\,dv}{g(v)} = \int f(x)\,dx}$

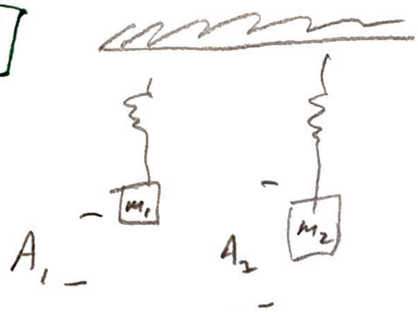
(b)

unfinished -5

(c)

2.15

10



If $E_{1Tot} = 2E_{2Tot}$ Find $\left(\frac{T_1}{T_2}\right)$

ratio of periods

Total energy $E_T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$

$\omega_i = \sqrt{\frac{k_i}{m_i}}$ $\Rightarrow \omega_i^2 m_i = k_i$

$\omega = f2\pi$

$\omega = \frac{2\pi}{T}$

use at $x = A_{max}$ $\dot{x} = 0$

so $E_{T_1} = \frac{1}{2}m(0)^2 + \frac{1}{2}k_1A_1^2 = \frac{1}{2}k_1A_1^2$

$E_{T_2} = \frac{1}{2}k_2A_2^2$

$E_{1T} = 2E_{2T}$ or $\frac{1}{2}(\omega_1^2 m_1)A_1^2 = 2\left(\frac{1}{2}\omega_2^2 m_2\right)A_2^2$

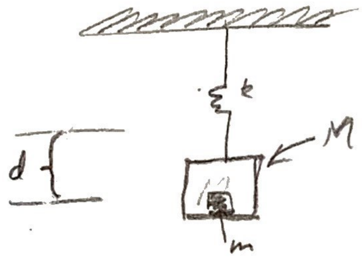
$\frac{\cancel{4\pi^2}}{T_1^2} m_1 A_1^2 = 2\left(\frac{\cancel{4\pi^2}}{T_2^2}\right)m_2 A_2^2$

$\frac{T_1^2}{T_2^2} = \frac{m_1 A_1^2}{2m_2 A_2^2}$

so $\boxed{\frac{T_1}{T_2} = \sqrt{\frac{m_1}{2m_2}}\,\frac{A_1}{A_2}}$ ✓

2.18

7/10



Find force of reaction between block and box as a function of time.

$d = ?$ when block just begin to leave the floor at top of an oscillation?

$F_{bl} = F_r - mg$

$F_r = F_{bl} + mg$

$F_r = m a_{bl} + mg$

for appropriate d

$$a_{bl} = a_{system} = \frac{-kx}{(M+m)}$$

$$(M+m)\ddot{x} = -kx \checkmark$$

$$(M+m)\ddot{x} + kx = 0 \checkmark \Rightarrow$$

$$x = A\cos\left(\sqrt{\frac{k}{M+m}}\,t + \theta_0\right)$$

So Force of reaction

$$\boxed{F_r = m\left(\frac{-kx}{M+m}\right) + mg}$$

at top $F_r = 0$

so $m\left(\frac{-k(d)}{M+m}\right) + mg = 0$

$$\frac{kd}{M+m} = g$$

$$\boxed{d = \frac{(M+m)g}{k}} \checkmark$$

$$F_r = \frac{kdm}{M+m}\cos\left(\sqrt{\frac{k}{M+m}}\right) + mg$$

-3

please see the key

such that

$\boxed{2.23}$ show that the driving frequency ω, $\exists$ $A_{(\omega)} = \frac{1}{2}A$ (w resonance) is $\omega_0 = \pm\gamma\sqrt{3}$.

$A \cong \dfrac{A_{max}\gamma}{\sqrt{(\omega_0-\omega)^2+\gamma^2}}$

8/10

Assuming its damped.

$A = \frac{1}{2}A_{max} \Rightarrow \frac{1}{2}\cancel{A_{max}} = \dfrac{\cancel{A_{max}}\gamma}{\sqrt{(\omega_0-\omega)^2+\gamma^2}}$

$4\gamma^2 = (\omega_0-\omega)^2+\gamma^2$

$|\omega_0-\omega| \cong \sqrt{3}\gamma$

QED

eq of motion $m\ddot{x}+b\dot{x}+kx = F_0\cos\omega t = F_{ox}$

let $z - iy = x$ then $m(\ddot{z}-i\ddot{y})+b(\dot{z}-i\dot{y})+k(z-iy) = F_0\cos\omega t = F_0(e^{i\omega t}-i\sin\omega t)$

$m\ddot{z}-mi\ddot{y}+b\dot{z}-bi\dot{y}+kz-kiy = F_0(e^{i\omega t}-i\sin\omega t)$

separate real $\boxed{m\ddot{z}+b\dot{z}+kz = F_0e^{i\omega t}}$

imag thing $mi\ddot{y}+bi\dot{y}+kiy = Fi\sin\omega t$

Solution: $z = z_{homogeneous} + z_{particular}$

homogeneous: $m\ddot{z}+b\dot{z}+kz = 0$

characteristic: $mq^2+bq+k=0$ $\quad q = \dfrac{-b\pm\sqrt{b^2-4mk}}{2m}$

so $\underline{z_{homo.} = C_1e^{q_1t}+C_2e^{q_2t}}$

particular: $\underline{z_p = z_0e^{i\omega t}}$ $\quad$ ~~ustl~~ $b(\dot{z}_p = i\omega z_0e^{i\omega t}) + (\ddot{z}_p = i^2\omega^2e^{i\omega t}z_0)m + kz_0e^{i\omega t} \stackrel{?}{=} F_0e^{i\omega t}$

$mi^2\omega^2z_0 + bi\omega z_0 + kz_0 \stackrel{?}{=} F_0$

$(-m\omega^2+bi\omega+k)z_0 = F_0$

so z_p is soln if $z_0 = \dfrac{F_0}{(-m\omega^2+bi\omega+k)(-m\omega^2-bi\omega+k)}$ rationalize denominator

$(m^2\omega^4)-m\omega^2ib(-m\omega^2k)+bi\omega^3m - b^2i^2\omega^2 - bi\omega k - m\omega^2k + kbi\omega + k^2)$

$z_0 = \dfrac{F_0(-m\omega^2-bi\omega+k)}{\{(k-m\omega^2)^2+b^2\omega^2\} = D}$

Amplitude $= \sqrt{Re(z_0)^2+Im(z_0)^2} = \sqrt{\left[\frac{F_0}{D}(-m\omega^2+k)\right]^2 + \frac{(-bi\omega)^2F_0^2}{D^2}} = \frac{F_0}{D}\sqrt{(-m\omega^2+k)^2+b^2\omega^2}$

$= \dfrac{F_0 D^{1/2}}{D} = \dfrac{F_0}{D^{1/2}}$

$$\boxed{A = \frac{F_0}{\sqrt{(k-m\omega^2)^2+b^2\omega^2}}}$$

(2.23 cont.)

if $\omega_0 = \sqrt{\frac{k}{m}}$ $\quad \gamma = \frac{c}{2m}$

then

$$A = \frac{F_0/m}{\sqrt{(\omega_0^2-\omega^2)^2 + 4\gamma^2\omega^2}}$$

resonant frequency $dA/d\omega = 0$

$$\frac{dA}{d\omega} \equiv 0 = \frac{F_0}{m}\frac{d(D^{-1/2})}{d\omega} = \frac{F_0}{m}\left(-\frac{1}{2}\right)D^{-3/2}\frac{dD}{d\omega} = \frac{-F_0}{2mD^{3/2}}\frac{d(\omega_0^2-\omega^2)^2+4\gamma^2\omega^2}{d\omega}$$

$$= \frac{-F_0}{2mD^{3/2}}\left[2(\omega_0^2-\omega^2)(-2\omega)+4\gamma^2 2\omega\right]$$

$$= \frac{2\omega F_0}{2mD^{3/2}}\left[\underbrace{-2(\omega_0^2-\omega^2)+4\gamma^2}_{0}\right]$$

so $2\gamma^2 + (-)(\omega_0^2-\omega_r^2) = 0$

$$\boxed{\omega_r^2 = \omega_0^2 - 2\gamma^2}$$

so ω_r^2

Now we are given $\frac{A(\omega)}{A(\omega_r)} = \frac{1}{2}$

$$\frac{\dfrac{F_0/m}{\sqrt{(\omega_0^2-\omega^2)^2+4\gamma^2\omega^2}}}{\dfrac{F_0/m}{\sqrt{(\omega_0^2-\omega_0^2+2\gamma^2)^2+4\gamma^2(\omega_0^2-2\gamma^2)}}} = \frac{1}{2} = \sqrt{\frac{4\gamma^4+4\gamma^2(\omega_0^2-2\gamma^2)}{(\omega_0^2-\omega^2)^2+4\gamma^2\omega^2}}$$

for $\omega = \omega_0 \pm \gamma\sqrt{3}$

$$\underline{4 \cdot 4\gamma^2(\omega_0^2-\gamma^2)} = (\omega_0^2-(\omega_0+\gamma\sqrt{3})^2)^2 + 4\gamma^2(\omega_0+\gamma\sqrt{3})^2$$

$16\gamma^2\omega_0^2 - 16\gamma^4$

$$= \omega_0^4 - 2\omega_0^2(\omega_0^2+2\gamma\omega_0\sqrt{3}+\gamma^2 3) + (\omega_0^2+2\gamma\omega_0\sqrt{3}+\gamma^2 3)^2 + 4\gamma^2\omega_0^2 + 2\cdot 4\gamma^3\omega_0\sqrt{3} + 4\gamma^4 3$$

$$= \omega_0^4 - \cancel{2\omega_0^4} - 4\gamma\omega_0^3\sqrt{3} - \boxed{\cancel{2\omega_0^2\gamma^2 3}} + \omega_0^4 + \omega_0^3 2\gamma\sqrt{3} + \boxed{\cancel{\omega_0^2\gamma^2 3}} + 2\gamma\omega_0^3\sqrt{3} + \boxed{4\gamma^2\omega_0^2 3} + \gamma^3 3\cdot 2\sqrt{3}\omega_0 + \boxed{\gamma^2\omega_0^2 3} + 2\gamma^3\omega_0 3\sqrt{3} + \gamma^4 \cdot 9$$

$$+ 4\gamma^2\omega_0^2 + 8\gamma^3\omega_0\sqrt{3} + 4\gamma^4 3$$

$= -\omega_0^4$

$$A(\omega) = A_{max} = \frac{F_0/m}{2\gamma\sqrt{\omega_0^2-\gamma^2}}$$

$$A(\omega_0+\gamma\sqrt{3}) = \frac{F_0/m}{\sqrt{(\omega_0^2-\underbrace{(\omega_0+\gamma\sqrt{3})}_{\omega}{}^2)^2+4\gamma^2(\omega_0+\gamma\sqrt{3})^2}}$$

$$\frac{A_{max}}{2} = A(\omega_0+\gamma\sqrt{3}) \Rightarrow 4\gamma\sqrt{\omega_0^2-\gamma^2} = \sqrt{\omega_0^4-2\omega_0^2\omega^2+\omega^4+4\gamma^2(\omega_0^2+2\gamma\sqrt{3}\omega_0+\gamma^2 3)}$$

$$16\gamma^2(\omega_0^2-\gamma^2) = \omega_0^4-2\omega_0^2(\omega_0^2+2\gamma\sqrt{3}\omega_0+\gamma^2 3) + (\omega_0^4+2\gamma\omega_0^3\sqrt{3}+\gamma^2\omega_0^2 3+2\gamma\sqrt{3}\omega_0^3$$

$\omega_0^2+2\gamma\sqrt{3}\omega_0+\gamma^2 3$

$$+4\gamma^2 3\omega_0^2+6\gamma^3\omega_0\sqrt{3}+\gamma^2 3\omega_0^2+2\gamma^3 3\cdot\sqrt{3}\omega_0+\gamma^4\cdot 9)$$

$$+4\gamma^2\omega_0^2+8\gamma^3\sqrt{3}\omega_0+4\gamma^4 3$$

$$\boxed{16\gamma^2\omega_0^2-16\gamma^4}$$

$$= \cancel{\omega_0^4} - \cancel{2\omega_0^4} - \cancel{4\gamma\sqrt{3}\omega_0^3} \boxed{-6\omega_0^2\gamma^2} + \cancel{\omega_0^4} + 2\gamma\omega_0^3\sqrt{3} + \boxed{\gamma^2\omega_0^2 3} + \cancel{2\gamma\sqrt{3}\omega_0^3} + \boxed{12\gamma^2\omega_0^2} + 4\sqrt{3}\gamma^3\omega_0 + \boxed{\gamma^2 3\omega_0^2}$$

$$+ 6\sqrt{3}\gamma^3\omega_0 + \gamma^4 9 + \boxed{4\gamma^2\omega_0^2} + 8\gamma^3\omega_0\sqrt{3} + 12\gamma^4$$

$$= \omega_0^2\gamma^2(\cancel{6}+\cancel{3}+12+\cancel{3}+4) + \gamma^3\omega_0(6\sqrt{3}+6\sqrt{3}+8\sqrt{3}) + \gamma^4(12+9)$$

$$\neq 16\omega_0^2\gamma^2 + 20\sqrt{3}\gamma^3\omega_0 + 21\gamma^4$$

SO $\omega_0+\gamma\sqrt{3}$ ≈ solves it.

only if γ is small then γ^3, γ^4 can be dropped.

Try $\omega_0 - \gamma\sqrt{3}$

$$\frac{A_{max}}{2} = \frac{F_0/m}{4\gamma\sqrt{\omega_0^2-\gamma^2}} = A(\omega_0-\gamma\sqrt{3}) = \frac{F_0/m}{\sqrt{\left(\omega_0^2-(\omega_0-\gamma\sqrt{3})^2\right)^2+4\gamma^2(\omega_0-\gamma\sqrt{3})^2}}$$

Square both sides, invert.

$$\underline{16\gamma^2(\omega_0^2-\gamma^2)} = \omega_0^2 - 2\omega_0^2(\omega_0-\gamma\sqrt{3})^2 + (\omega_0-\gamma\sqrt{3})^4 + 4\gamma^2(\omega_0^2 - 2\gamma\omega_0\sqrt{3} + \gamma^2 3)$$

$$= \omega_0^2 - 2\omega_0^2(\omega_0^2 - 2\omega_0\gamma\sqrt{3} + \gamma^2 3) + \left(\omega_0^4 - 2\omega_0^3\gamma\sqrt{3} + \omega_0^2\gamma^2 3 - 2\omega_0^3\gamma\sqrt{3} + 4\omega_0^2\gamma^2\cdot 3 - 2\omega_0\gamma^3\sqrt{3}\cdot 3\right.$$
$$(\omega_0^2 - 2\omega_0\gamma\sqrt{3} + \gamma^2 3)$$

$$\left. + \omega_0^2\gamma^2 3 + \gamma^2 3(-2\omega_0\gamma\sqrt{3}) + \gamma^4\cdot 9\right) + 4\gamma^2\omega_0^2 - 8\gamma^3\omega_0\sqrt{3} + 4\gamma^4 3$$

$$= \cancel{\omega_0^2} - \cancel{2\omega_0^4} + \boxed{\cancel{4\omega_0^3\gamma\sqrt{3}}} - 2\omega_0^2\gamma^2 3 + \cancel{\omega_0^2} \boxed{\cancel{-2\omega_0^3\gamma\sqrt{3}}} + \omega_0^2\gamma^2 3 \boxed{\cancel{-2\omega_0^3\gamma\sqrt{3}}} + 12\omega_0^2\gamma^2$$
$$- 6\omega_0\gamma^3\sqrt{3} + \omega_0^2\gamma^2 3 - 6\gamma^3\omega_0\sqrt{3} + \gamma^4\cdot 9 + 4\gamma^2\omega_0^2 - 8\gamma^3\omega_0\sqrt{3} + 12\gamma^4$$

$$= \gamma^2\omega_0^2(-6+3+12+3+4) + \gamma^3\omega_0(-6\sqrt{3}-6\sqrt{3}-8\sqrt{3}) + 21\gamma^4$$

$$= \underline{16\gamma^2\omega_0^2} - 20\sqrt{3}\gamma^3\omega_0 + 21\gamma^4$$

again for small γ the γ^3 & γ^4 can be dropped.

please see the key