7.1 (a) Chp 7 Rigid Bodies – Planar Motion

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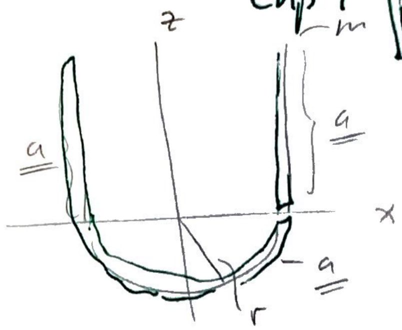
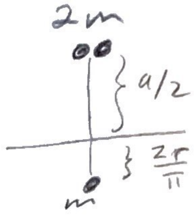
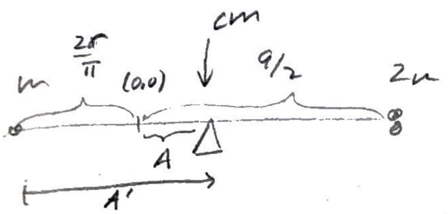


$\equiv$



The book def. c.m. as concentrated mass at that point.

The book also brings out that the moments of a rigid body $= 0$ about the cm.
I'll seek an 'A' so that Σ moments $= 0$



$$(m)\left(-\frac{2r}{\pi}+A\right) = \left(\frac{a}{2}-A\right)2m \qquad \pi r = a \qquad r = \frac{a}{\pi}$$

so

$$\left(-\frac{2a}{\pi^2}+A\right) = \left(\frac{a}{2}-A\right)2$$

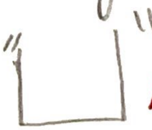
$$-\frac{2a}{\pi^2}+A+2A = a \longrightarrow 3A = a + \frac{2a}{\pi^2}$$

$$A = \frac{a}{3} + \frac{2}{3}\frac{a}{\pi^2} = \frac{a}{3}\left(1+\frac{2}{\pi^2}\right)$$

from origin

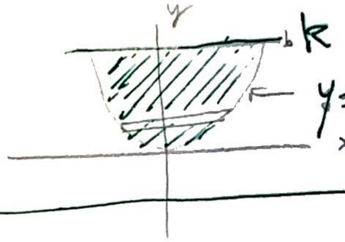
From vertex:

$$A' = -\frac{2a}{\pi^2} + A = -\frac{2a}{\pi^2} + \frac{a}{3} + \frac{2a}{\pi^2} = \frac{a}{3} + \frac{4a}{\pi^2}$$

WAIT It Just dawned on me that they want $\Rightarrow$ this kind of "" yes

(see back please)

7.1 cont (b)



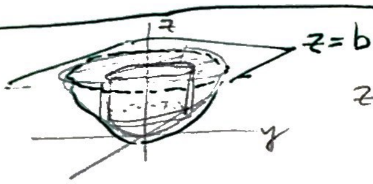
By symmetry c.m. will be on y axis:

$$y_{cm} = \frac{\rho \int_A y\, dA}{\rho \int_A dA} = \frac{\int_0^{+(b/a)^{1/2}} (ax^2)[2x\, dy]}{\int_0^{+(b/a)^{1/2}} 2x\, dy}$$

by $y = ax^2$
$dy = a2x\,dx$

$$y_{cm} = \frac{\int_0^{(b/a)^{1/2}} (ax^2)\, 2x(a2x\,dx)}{\int_0^{(b/a)^{1/2}} 2x\, a2x\, dx} = \frac{a \left.\frac{x^5}{5}\right|}{\left.\frac{x^3}{3}\right|} = \frac{\frac{a}{5}\frac{b^{5/2}}{a^{5/2}}}{\frac{b^{3/2}/a^{3/2}}{3}} = \boxed{\frac{3}{5} b = y_{cm}}$$

(c)



$z = ay^2$

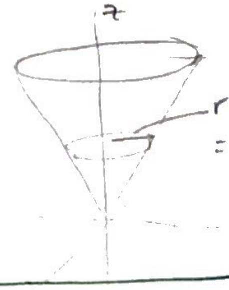
Again symmetry says c.m. will be on z axis.

using cylindrical shells $dV = dy\, 2\pi y(b - ay^2)$

$$z_{cm} = \frac{\rho \int_0^{(b/a)^{1/2}} ay^2\, 2\pi y\,(b - ay^2)\, dy}{\rho \int_0^{(b/a)^{1/2}} y(b - ay^2)\, dy} = a \frac{\int (y^3 b - ay^5)\, dy}{\int (yb - ay^3)\, dy} = a \frac{\left. \frac{y^4}{4} b - \frac{ay^6}{6} \right|_0^{(b/a)^{1/2}}}{\left. \frac{by^2}{2} - \frac{ay^4}{4} \right|_0^{(b/a)^{1/2}}}$$

$$= a \frac{\frac{\frac{b^2}{a^2} b}{4} - \frac{a^2 b^3/a^3}{6}}{\frac{b}{2}\left(\frac{b}{a}\right) - \frac{a}{4}\left(\frac{b^2}{a^2}\right)} = \frac{b\left(\frac{1}{4} - \frac{1}{6}\right)}{\left(\frac{1}{2} - \frac{1}{4}\right)} = b\left(\frac{1}{12}\right)\left(\frac{4}{1}\right) = \boxed{b\,\frac{2}{3} = z_{cm}}$$

7.1 cont (d)



$z = \frac{h}{a} y \qquad dz = \frac{h}{a} dy$

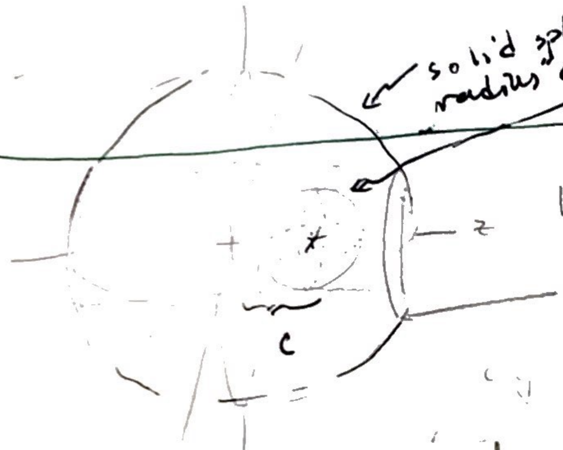
$dV = \pi r^2 dz = \pi y^2 dz = \pi \frac{a^2}{h^2} z^2 dz$

CM

$$z_{cm} = \frac{\int_0^h z^3 \frac{a^2}{h^2} dz}{\int_0^h z^2 \frac{a^2}{h^2} dz} = \frac{\frac{z^4}{4}\frac{a^2}{h^2}\Big|_0^h}{\frac{z^3}{3}\frac{a^2}{h^2}\Big|_0^h} = \boxed{\frac{3}{4} h = z_{cm}}$$

7.2

5/5



By symmetry the c.m. will be on z axis.

$$z_{cm} = \frac{1}{M} \int_{O} z\, dm = \frac{1}{M}\Big[\underbrace{\overset{\text{Big}}{\int_{O} z\, dm} - \overset{\text{Little}}{\int_{o} z\, dm}}\Big]$$

$\int_o z\,dm = M_0 z_{cm_0} = c$

$M_O z_{cm_O} = 0$

CM

$$z_{cm} = \frac{1}{M}(0 - M_0 c)$$

$$= \frac{\rho\, (-\frac{4}{3}\pi b^3 c)}{\rho \frac{4}{3}\pi (a^3 - b^3)} = \boxed{\frac{-b^3 c}{a^3 - b^3} = z_{cm}}$$

$\boxed{7.3}$ (a) Find $\boxed{\text{moment of inertia}}$ about axis of symmetry:

$\boxed{r = \frac{a}{\pi}}$

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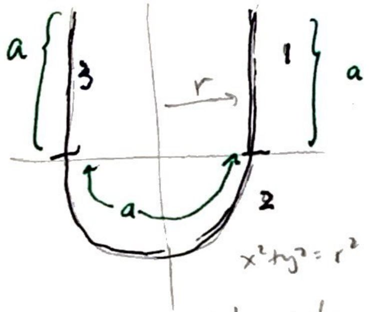


$x^2+y^2=r^2$

$xdx = -ydy$

$$I = \sum_i \int R^2 \, dm_i$$

$$\rho = \frac{m}{3a}$$

- $I_1 = \left(\frac{m}{3}\right) r^2 = \frac{ma^2}{3\pi^2}$
- $I_3 = \left(\frac{m}{3}\right) r^2 = \frac{ma^2}{3\pi^2}$
- $I = 2\rho \int_0^r x^2 \left(\frac{r^2}{r^2-x^2}\right)^{\frac{1}{2}} dx = 2\rho\left(\frac{r^2}{2}\sin^{-1}\frac{x}{r} - \frac{1}{2}x\sqrt{r^2-x^2}\right)\Big|_0^r$

$dm = \rho \, dl = \rho\left(dx^2 + dy^2\right)^{\frac{1}{2}}$ $\quad \frac{x^2 dx^2}{y^2}$

$$= \rho\left(1 + \frac{x^2}{r^2-x^2}\right)^{\frac{1}{2}} dx$$

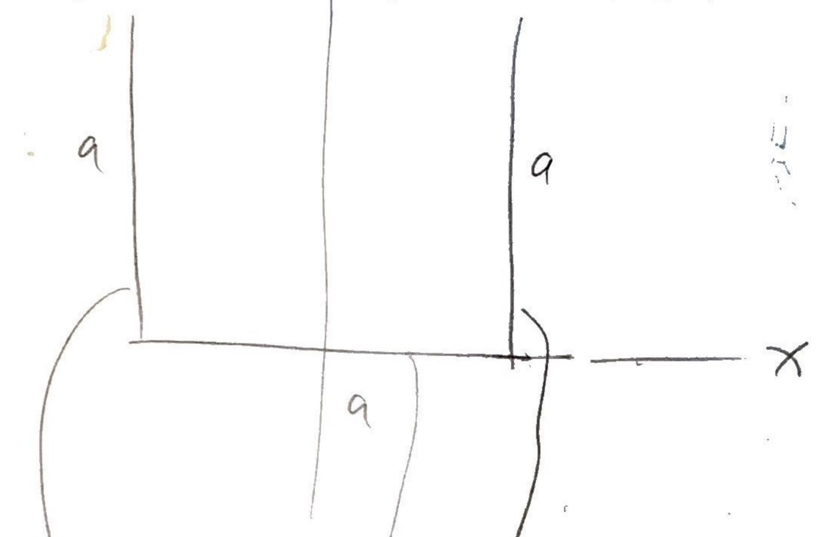
$$= 2\rho\left[\frac{r^2}{2}\left(\sin^{-1}1 - \sin^{-1}0\right)\right]$$

$$= \frac{2\rho r^2}{2}\left[\frac{\pi}{2}\right] = \frac{\pi \rho r^2}{2} = \frac{a^2\rho}{2\pi}$$

- $I_1 + I_3 + I_2 = \frac{2}{3}\frac{ma^2}{\pi^2} + \frac{am}{6\pi}$

see back of page 2

7.3(a)



$I = \frac{m}{3}\left(\frac{a}{2}\right)^2$

$I = \frac{m}{3}\left(\frac{a}{2}\right)^2$

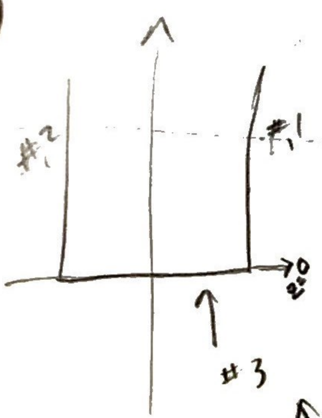
$I = \left(\frac{m}{3}\right)\frac{a^2}{12}$

$I_t = \frac{2}{3}\frac{a^2}{4} + \frac{ma^2}{36}$

$$\underline{\underline{I}} = \underline{\underline{\frac{7}{36} ma^2}}$$

7.1(a)

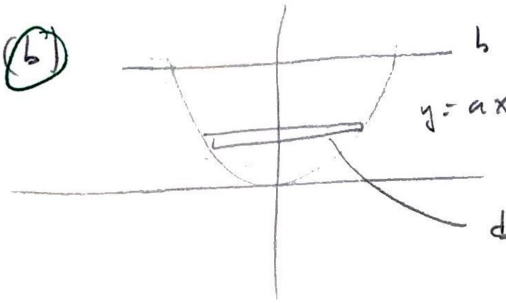
(golly how simple)



$$y_{cm} = \frac{\overbrace{\sum my}^{\text{moments}}}{\sum m} = \frac{2\overbrace{\left[\frac{m}{3}\left(\frac{a}{2}\right)\right]}^{\#1,2} + \frac{m}{3}(0)}{m} = \underline{\underline{\frac{a}{3}}}$$

No contribution b c.m. on z

$\boxed{7.3}$ (b)



$dI = \frac{1}{12} dm (2x)^2$ $\quad dm = \rho dA = 2x\rho\, dy$

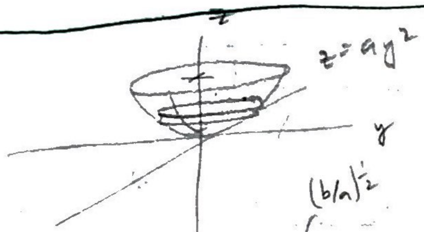
$dI = \frac{1}{12} 4x^2\, 2x\rho\, dy$ but $dy = a2x\,dx$ so

$$\underline{I} = \frac{4\rho a}{3}\int_0^{(b/a)^{1/2}} x^4\,dx = \frac{4\rho a}{3}\frac{x^5}{5}\Big|_0^{(b/a)^{1/2}} = \underline{\rho\,\frac{4}{3}a\,\frac{b^{5/2}/a^{5/2}}{5}}$$

but from 7.1 (b) $\rho = \frac{3}{4} m \frac{a^{1/2}}{b^{3/2}}$

$$\underline{I} = \frac{3}{4} m \frac{4}{3} a \frac{a^{1/2}}{b^{3/2}} \frac{b^{5/2}}{a^{5/2}} \frac{1}{5} = \boxed{\frac{1}{5} m \frac{b}{a} = I}$$

(c)



$dI = \frac{1}{2} dm\, y^2$

$dm = dz\,\pi y^2 \rho$

$$I = \int_0^{(b/a)^{1/2}} \frac{1}{2} y^2\, \pi y^2 \rho\; 2ay\,dy$$

$$= \pi\rho a \int y^5\,dy = \pi\rho a \frac{y^6}{6}\Big|_0^{(b/a)^{1/2}} = \frac{\pi\rho a\, b^3/a^3}{6}$$

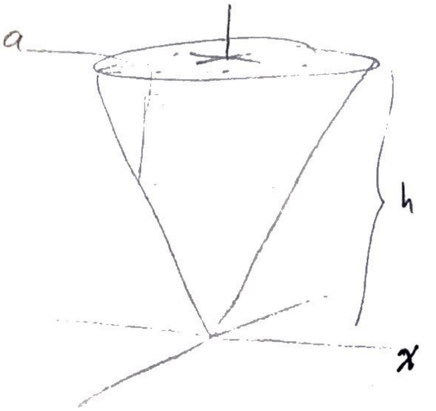
$I = \frac{\pi a}{6} y^6 \left(\frac{2m a}{\pi y^4}\right)$

$I = \frac{m}{3} y^2$

but $\rho = \frac{m}{V} = \frac{m}{\int_0^{(b/a)^{1/2}} \pi y^2\, 2ay\,dy} = \frac{m}{2\pi a \frac{y^4}{4}\Big|_0^{(b/a)^{1/2}}} = \frac{4m}{2\pi a}\frac{a^2}{b^2}$

so $I = \frac{\pi b^3/a^2}{6}\left(\frac{2m}{\pi}\frac{a}{b^2}\right) = \underline{\frac{1}{3} m \frac{b}{a}}$

7.3 (d)



$$dI = dm\, r^2$$

$$dm = \left(h - \frac{h}{a}x\right)(2\pi x)\, dx\, \rho$$

$$I = \int_0^a x^2\left(h - \frac{h}{a}x\right) 2\pi x\, dx\, \rho$$

$$= \rho 2\pi h \int x^3 dx - \frac{2\pi h \rho}{a}\int x^4 dx$$

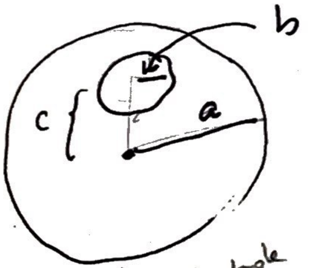
$$= 2\pi h \rho a^4 \left[\frac{1}{4} - \frac{1}{5}\right] = \frac{2\pi \rho a^4}{20} = \frac{\pi h a^4 \rho}{10}$$

$$\rho = \frac{m}{V} = \frac{m}{\frac{\pi a^2 h}{3}} \implies \underline{\underline{I}} = \frac{3\pi h a^4 m}{10\,\pi a^2 h} = \underline{\underline{\frac{3}{10} m a^2}}$$

7.4

Find the moment of inertia for prob 7.2,

5/5



$$I = I_{sphere} - I_{little}$$

- $I_{little} = \frac{2}{5} m b^2$
- $I_B = \frac{2}{5} M a^2$ $\quad I_T = I_B - I_l = \left[\frac{2}{5} M a^2 - \frac{2}{5} m b^2\right]$

let $\underline{\underline{\mu = M - m}}$ (μ: w/ hole, M: w/o hole, m: the hole)

$$I = \frac{2}{5} M a^2 - [M - \mu]\left[\frac{2}{5} b^2\right]$$

but $\frac{\mu}{M} = \frac{V_B - V_l}{V_B} = 1 - \frac{V_l}{V_B} = 1 - \frac{b^3}{a^3}$

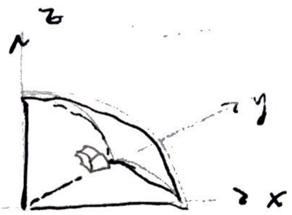
$$I = \frac{2}{5}\mu a^2\left(\frac{1}{1 - b^3/a^3}\right) - \mu\left[\frac{b^3/a^3}{1 - b^3/a^3}\right]\left[\frac{2b^2}{5}\right]$$

$$\boxed{I = \frac{\mu}{a^3 - b^3}\left[\frac{2}{5}(a^5 - b^5)\right]}$$

μ: total mass present.

7.5

Octant of sphere

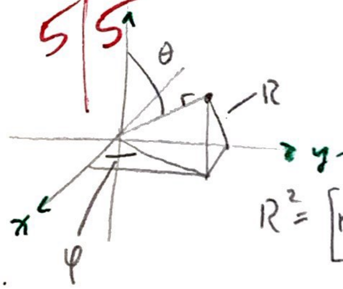


$dm = \rho dV$

$dV = dx\,dy\,dz$

$= h_1 h_2 h_3\, du_1\, du_2\, du_3$

$= r^2 \sin\theta\, dr\, d\theta\, d\varphi$



5/5

$$I = \int_V R^2 dM$$

$$R^2 = \left[r^2 s^2\left(\tfrac{\pi}{2}-\theta\right) = r^2 c^2\theta\right] + \left[r^2 c^2\left(\tfrac{\pi}{2}-\theta\right)\sin^2\varphi = r^2 s^2\theta\, s^2\varphi\right]$$

$$I = \int_V r^2\left[c^2\theta + s^2\theta s^2\varphi\right]\rho r^2 s\theta\, dr\, d\theta\, d\varphi$$

$$I = \rho \int_0^{\pi/2}\int_0^{\pi/2}\int_0^{a}\left(r^4 c^2\theta s\theta + r^4 s^3\theta s^2\varphi\right) dr\, d\theta\, d\varphi$$

$$= \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \frac{r^5}{5}\Big|_0^a \left(c^2\theta s\theta + s^3\theta s^2\varphi\right) d\theta\, d\varphi$$

$$= \frac{\rho a^5}{5}\int_0^{\pi/2}\left[-\frac{\cos^3\theta}{3} + s^2\varphi\left(-\frac{s^2\theta c\theta}{3} + \frac{2}{3}(-\cos\theta)\right)\right]_0^{\pi/2} d\varphi$$

$$= \frac{\rho a^5}{5}\left[\frac{1}{3}\frac{\pi}{2} + \frac{2}{3}\left[\frac{u}{2} - \frac{\sin 2\theta}{4}\right]\Big|_0^{\frac{\pi}{2}}\right] = \frac{\rho a^5}{5}\left[\frac{\pi}{6}+\frac{\pi}{6}\right] = \frac{\pi a^5 \rho}{15}$$

but $\rho = \dfrac{m}{\left(\frac{4}{3}\pi a^3\right)\frac{1}{8}} = \dfrac{6m}{\pi a^3}$

$$\Rightarrow \underline{\underline{I}} = \frac{\pi a^5}{15}\left(\frac{6m}{\pi a^3}\right) = \boxed{\underline{\frac{2}{5}ma^2} = I}$$

please see the key

$\boxed{7.6}$ $\frac{1}{2}$ circle wire hangs from peg. About to slip. Find μ_{static}

$x^2+y^2=a^2$

$\Rightarrow \frac{dy}{dx} = \frac{x}{y} = \frac{1}{\tan\alpha}$

• geometry

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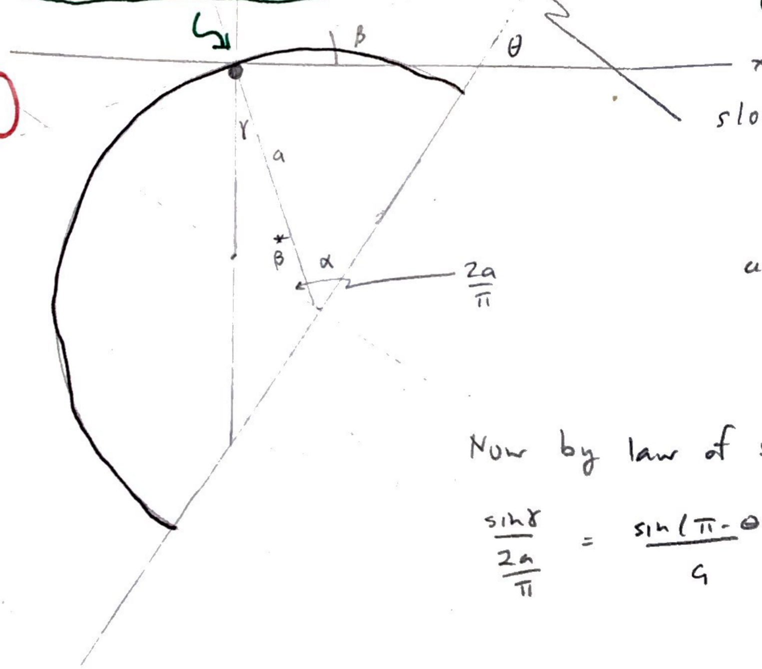


slope $\cot\alpha = \tan\beta$

$$\frac{\cos\alpha}{\sin\alpha} = \frac{\sin\beta}{\cos\beta}$$

and so $\cos\alpha\cos\beta - \sin\alpha\sin\beta = 0$

$\cos(\alpha+\beta) = 0$

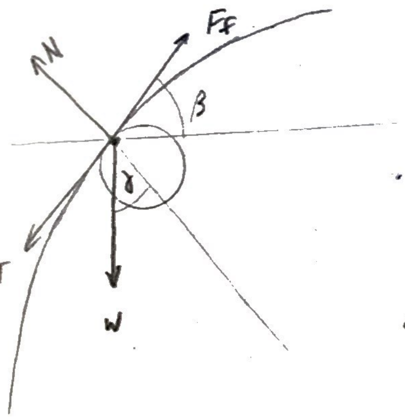
$\Rightarrow \underline{\alpha+\beta = \frac{\pi}{2}}$ *

Now by law of sines

$$\frac{\sin\gamma}{\frac{2a}{\pi}} = \frac{\sin(\pi-\theta)}{a} \Rightarrow \sin\gamma = \frac{2}{\pi}\sin\theta$$

$$\boxed{\gamma = \sin^{-1}\left(\frac{2}{\pi}\sin\theta\right)}$$

• So much for math. Newton's Law now...



$W\sin\gamma = T$

$W\cos\gamma = N$

$F_f = N\mu = mg\cos\gamma\,\mu$

at limiting equilibrium

$F_f = T$

$\mu mg\cos\gamma = mg\sin\gamma$

or $\tan\gamma = \mu$

$$\boxed{\tan\left(\sin^{-1}\left(\frac{2}{\pi}\sin\theta\right)\right) = \mu}$$

$$\mu = \frac{\frac{2}{\pi}\sin\theta}{\sqrt{1-\frac{4}{\pi^2}\sin^2\theta}}$$

good job. keep it up!

$\boxed{7.9}$ Prove that if $\sum F_i = 0$ (trans equilib) and $\sum \vec{r}_i \times \vec{F}_i = 0$ (Rot. equilib) about O then its in rot. eq. about O'

5/5

let $\vec{r}_i = \vec{r}_i' + \vec{R}$

Rot. eq. $\Rightarrow 0 = \underbrace{\sum \vec{r}_i \times \vec{F}_i}_{\text{given}} = \sum(\vec{r}_i' + \vec{R}) \times \vec{F}_i$

$= \sum \vec{r}_i' \times \vec{F}_i + \sum \vec{R} \times \vec{F}_i = 0$

$= \sum \vec{r}_i' \times \vec{F}_i + \vec{R} \times (\sum \vec{F}_i) = 0$

but $\sum F_i = 0$ so $\boxed{\sum \vec{r}_i' \times \vec{F}_i = 0}$ ↑ from O'

$\boxed{7.11i}$ $\boxed{\text{physical pendulum}}$

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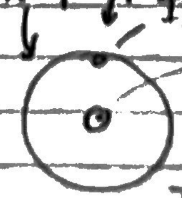


hoop, pivot I_0

$I = ma^2$

by ∥ axis

$I_0 = ma^2 + (I) = 2ma^2$

using $T = 2\pi\sqrt{\frac{I}{mgl}}$

we get $T = 2\pi\sqrt{\frac{2ma^2}{mga}}$

$\boxed{T = 2\pi\sqrt{\frac{2a}{g}}}$

now use ⊥ axis theorem

$I = ma^2$

$I_z = I_x + I_y$ for lamina

but by symmetry $I_x = I_y$

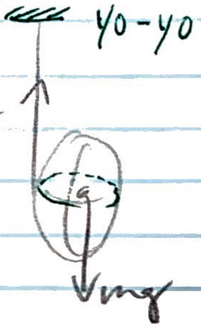
$ma^2 = 2I_x \quad I_x = \frac{ma^2}{2}$

use ∥ axis theorem

$I_0 = ma^2 + \frac{ma^2}{2}$

$T = 2\pi\left(\frac{3/2\, ma^2}{mga}\right)^{\frac{1}{2}} = \boxed{2\pi\left(\frac{3}{2}\frac{a}{g}\right)^{\frac{1}{2}}}$

7.13 — What is the acceleration of the center of the ~~sphere~~ yo-yo



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① $T - mg = m\ddot{y}$

② $\vec{N} = \dfrac{d\vec{L}}{dt}$

Now $N = aT$

$L = I_{cm}\omega$

$\dot{L} = I_{cm}\dot{\omega}$

No slipping says

$-\dot{\omega} a = \ddot{y}$

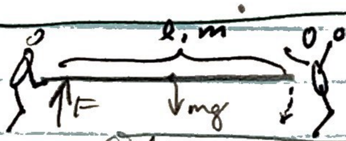
so $T = \dfrac{N}{a} = \dfrac{\dot{L}}{a} = \dfrac{I_{cm}(-\dot{\omega})}{a} = \dfrac{I_{cm}\left(\frac{\ddot{y}}{a}\right)}{a}$

$= \dfrac{I_{cm}\ddot{y}}{a^2}$

① becomes $\dfrac{I_{cm}\ddot{y}}{a^2} - m\ddot{y} = mg \Rightarrow \ddot{y}\left[\dfrac{\frac{2}{5}ma^2}{a^2} - 1\right] = g$

or $\boxed{\ddot{y} = -g\dfrac{5}{3}}$ ✓

7.14



(a) Show that the weight @ other end is initially $\dfrac{mg}{4}$

. Torque: $N_{cm} = F\dfrac{l}{2}$ → $F = \dfrac{2}{l}\dfrac{1}{12}ml\left(\dfrac{3g}{2}\right)$ — a *

$= \dot{L} = \dfrac{1}{12}ml^2\dot{\omega}$ so $F = \dfrac{m}{6}\dfrac{3}{2}g = \boxed{\dfrac{mg}{4} = F}$ ✓

10

(b) Acceleration: $\Sigma N = \dfrac{l}{2}mg$

$I\dot{\omega} = \dfrac{1}{3}ml^2\dot{\omega}$

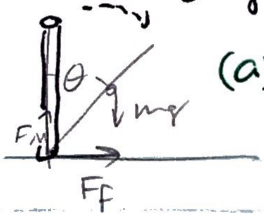
$\dfrac{1}{3}l^2\dot{\omega} = \dfrac{l}{2}g$

so $\dfrac{3}{2}\underbrace{(l\dot{\omega})}_{a} = g$ $\boxed{a = \dfrac{3g}{2}}$ * ✓

7.19

6

Long uniform rod starts to fall over.



(a) Find $\vec{F}_N$ and $\vec{F}_{Tan}$ reaction of the floor as functions of θ.

(a)

eq. of motion: ① $m\ddot{x} = -mg\sin\theta + F_F$

② $m\ddot{y} = -mg\cos\theta + F_N$

② becomes, after using $x = \frac{l}{2}\sin\theta$, $y = \frac{l}{2}\cos\theta$

$\ddot{x} = \frac{l}{2}\sin\theta\,\ddot{\theta}$, $\ddot{y} = \frac{-l}{2}\cos\theta\,\ddot{\theta}$

$$m\left(\frac{-l}{2}\cos\theta\,\ddot{\theta}\right) = -mg\cos\theta + F_N$$

Now torque:

$$\vec{N} = \frac{d\vec{L}}{dt} = |\vec{r}\times m\vec{g}| = \frac{l}{2}\sin\theta\, mg = I_{cm}\dot{\omega}$$

$\frac{ml^2}{3}$, $\ddot{\theta}$

so $\ddot{\theta} = \frac{\cancel{6}^{3}\sin\theta}{2l}g$

now ① becomes

$$F_N = -\frac{ml}{2}g\cos\theta\left(\frac{6\sin\theta}{l}\right) + mg\cos\theta$$

vertical: $\boxed{F_N = mg\left[\cos\theta\,(1-3\sin\theta)\right]}$ ×

$\frac{1}{4}mg\,(3\cos\theta-1)^2$

horizontal use #1 eq.

$m\ddot{x} = ~~F_x = -mg\sin\theta + \mu F_N~~$

was to find θ when slipping starts.

$$\boxed{F_x = -mg\sin\theta + \mu\left[mg\cos\theta\,(1-3\sin\theta)\right]}$$ ×

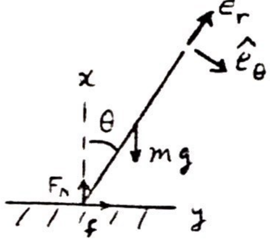
↓

(b) ? missing

$\frac{3}{4}mg\sin\theta\,(3\cos\theta-2)$

please see the key

7.19)



(a) While the rod is not slipping, it rotates about the fixed point on the floor as a fixed axis, and $f < \mu F_N$.

When $|f| = \mu F_N$, the rod begins to slip. Then $f = \mu F_N$ from that point on.

<u>While the rod is not slipping</u>

Take torque about axis of rotation:

$$N = mg\frac{\ell}{2}\sin\theta = I\dot{\omega} = I\ddot{\theta}$$

$$I = \frac{m\ell^2}{3}$$

$$\Rightarrow \ddot{\theta} = \frac{3}{2}\frac{g}{\ell}\sin\theta$$

To get the reaction forces, we use $\Sigma\vec{F} = m\vec{a}_{cm}$.

$$\Sigma\vec{F} = (f\sin\theta + F_N\cos\theta - mg\cos\theta)\hat{e}_r + (f\cos\theta - F_N\sin\theta + mg\sin\theta)\hat{e}_\theta$$

$$\vec{a}_{cm} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

$$r = \frac{\ell}{2} \Rightarrow \dot{r} = \ddot{r} = 0$$

$$\Rightarrow f\sin\theta + F_N\cos\theta - mg\cos\theta = -\frac{m\ell}{2}\dot{\theta}^2$$

$$f\cos\theta - F_N\sin\theta + mg\sin\theta = \frac{m\ell}{2}\ddot{\theta} = \frac{3}{4}mg\sin\theta \quad (1)$$

To get $\dot{\theta}^2$, we take $\ddot{\theta} = \frac{d\dot{\theta}}{dt} = \frac{d\dot{\theta}}{d\theta}\frac{d\theta}{dt} = \dot{\theta}\frac{d\dot{\theta}}{d\theta} = \frac{d}{d\theta}\left(\frac{1}{2}\dot{\theta}^2\right)$

$$\Rightarrow \int_0^{\frac{1}{2}\dot{\theta}^2} d\left(\frac{1}{2}\dot{\theta}^2\right) = \int_0^\theta \frac{3}{2}\frac{g}{\ell}\sin\theta\, d\theta$$

$$\Rightarrow \frac{1}{2}\dot{\theta}^2 = -\frac{3}{2}\frac{g}{\ell}\cos\theta + \frac{3}{2}\frac{g}{\ell}$$ (Same result from conservation of energy)

$$\Rightarrow f\sin\theta + F_N\cos\theta - mg\cos\theta = \frac{3}{2}mg\cos\theta - \frac{3}{2}mg \quad (2)$$

(7.14 continued)

$$(1): \quad f\cos\theta - F_N\sin\theta = -\frac{mg}{4}\sin\theta$$

$$(2): \quad f\sin\theta + F_N\cos\theta = \frac{mg}{2}(5\cos\theta - 3)$$

$$(1)\times\cos\theta + (2)\times\sin\theta \Rightarrow f = -\frac{mg}{4}\sin\theta\cos\theta + \frac{mg}{2}\sin\theta(5\cos\theta-3)$$

$$\Rightarrow f = \frac{mg}{4}\sin\theta\,(10\cos\theta - 6 - \cos\theta)$$

$$\Rightarrow f = \frac{1}{4}mg\sin\theta\,(9\cos\theta - 6)$$

$$\Rightarrow \underbrace{f = \frac{3}{4}mg\sin\theta\,(3\cos\theta - 2)}.$$

$$-(1)\times\sin\theta + (2)\times\cos\theta \Rightarrow F_N = \frac{1}{4}mg\sin^2\theta + \frac{5}{2}mg\cos^2\theta - \frac{3}{2}mg\cos\theta$$

$$\Rightarrow F_N = \frac{1}{4}mg + \frac{9}{4}mg\cos^2\theta - \frac{6}{4}mg\cos\theta$$

$$= \frac{1}{4}mg\,(9\cos^2\theta - 6\cos\theta + 1)$$

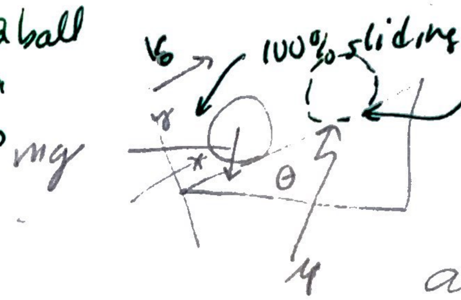
$$\Rightarrow \underbrace{F_N = \frac{1}{4}mg\,(3\cos\theta - 1)^2}.$$

(b) The rod begins to slip when $|f| = \mu F_N$

$$\Rightarrow \underbrace{\left|3\sin\theta\,(3\cos\theta - 2)\right| = \mu\,(3\cos\theta - 1)^2}$$

It will slip to left in my drawing (backwards), if $\cos\theta < \frac{2}{3}$ at initial slip point; otherwise it will slip to right (forward).

$\boxed{7.21}$ Kick a ball up a ramp

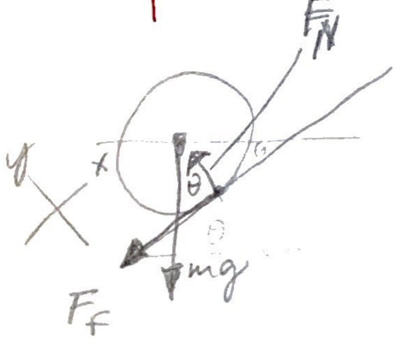


Find $x(t)$

& x when pure rolling

assume $\mu > \frac{2}{7}\tan\theta$

7



$$m\ddot{x} = -F_f - mg\sin\theta \qquad F_f = \mu N$$

$$m\ddot{y} = F_N - mg\cos\theta \qquad \ddot{y} = 0 \Rightarrow F_N = mg\cos\theta$$

$$\cancel{m}\ddot{x} = -\mu \cancel{m}g\cos\theta - \cancel{m}g\sin\theta = -\cancel{m}g(\mu\cos\theta + \sin\theta)$$

$$I_{cm}\dot{\omega} = N = aF_f = a\mu mg\cos\theta$$

$$\dot{\omega} = \frac{a\mu mg\cos\theta}{I} \qquad I = mk^2$$

$$= \frac{a\mu g\cos\theta}{k^2}$$

integrate

$$\omega = \left(\frac{a\mu g}{k^2}\cos\theta\right)t$$

$$\dot{x} = -\cancel{m}g(\mu\cos\theta + \sin\theta)t + v_0$$

$$\boxed{x = -\frac{\cancel{m}g(\mu\cos\theta + \sin\theta)t^2}{2} + v_0 t + \cancel{x_0}^{\,0}} \quad 1$$

$$\frac{\omega k^2}{a\mu g\cos\theta} = \frac{\dot{x} - v_0}{-mg(\mu c\theta + s\theta)}$$

$$a\omega\left(\frac{-k^2 mg(\mu\cos\theta + s\theta)}{a^2\mu g\cos\theta}\right) + v_0 = \dot{x}$$

for pure rolling

$$\dot{x} = a\omega$$

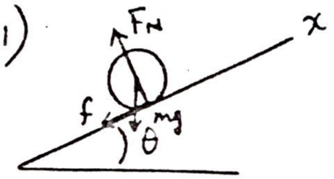
$$x = a\omega t + x_0$$

$$a\omega(\gamma) + v_0 = a\omega$$

$$v_0 = a\omega(1-\gamma)$$

unfinished

$$\rightarrow \quad x = \frac{2v_0^2}{g}\,\frac{\sin\theta + 6\mu\cos\theta}{(2\sin\theta + 7\mu\cos\theta)^2}$$

7.21) 

Initial speed v_0.

$f \leq \mu F_N$. $\qquad \mu > \frac{2}{7}\tan\theta$.

Let x be along the incline, positive upwards.

$\sum \vec{F} = m\vec{a}_{cm} \Rightarrow -\mu mg\cos\theta - mg\sin\theta = m\ddot{x}$

since $f = \mu mg\cos\theta$ prior to rolling without slipping.

$\Rightarrow \quad \ddot{x} = -\mu g\cos\theta - g\sin\theta$.

$\Rightarrow \quad \dot{x} = v_0 - g(\sin\theta + \mu\cos\theta)t$

$\Rightarrow \quad \underline{x = v_0 t - \frac{1}{2}g(\sin\theta + \mu\cos\theta)t^2} \qquad (x_0 = 0)$

Rolling begins (without slipping) when $\dot{x} = \omega a$.

$N = af = I_c\dot{\omega}$

$\Rightarrow \quad a\mu mg\cos\theta = \frac{2}{5}ma^2\dot{\omega}$

$\Rightarrow \quad \dot{\omega} = \frac{5}{2}\frac{\mu g\cos\theta}{a}$

$\omega_0 = 0 \quad \Rightarrow \quad \omega = \frac{5}{2}\frac{\mu g\cos\theta}{a}t$.

So pure rolling begins when t is given by $\quad \omega a = \dot{x}$

$\omega a \quad = V_0 - g\,(s\theta + \mu c(\theta))t = \dot{x}$

$\frac{5}{2}\mu g\cos\theta\, t = v_0 - g(\sin\theta + \mu\cos\theta)t$

$\Rightarrow \quad t = \frac{v_0}{g}\,\frac{1}{\sin\theta + \frac{7}{2}\mu\cos\theta}$

$\Rightarrow \quad x = \frac{v_0^2}{g}\frac{(\sin\theta + \frac{7}{2}\mu\cos\theta)}{(\sin\theta + \frac{7}{2}\mu\cos\theta)^2} - \frac{g}{2}(\sin\theta + \mu\cos\theta)\frac{v_0^2}{g^2}\frac{1}{(\sin\theta + \frac{7}{2}\mu\cos\theta)^2}$

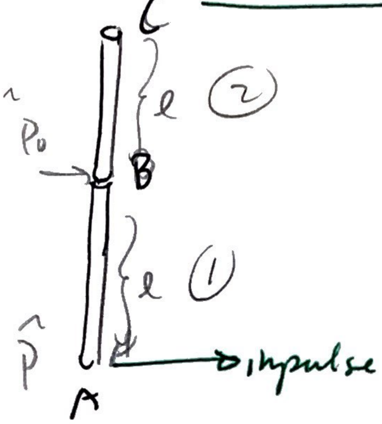
$= \frac{v_0^2}{g}\frac{(\sin\theta + \frac{7}{2}\mu\cos\theta - \frac{1}{2}\sin\theta - \frac{1}{2}\mu\cos\theta)}{(\sin\theta + \frac{7}{2}\mu\cos\theta)^2} = \frac{2v_0^2}{g}\frac{(\sin\theta + 6\mu\cos\theta)}{(2\sin\theta + 7\mu\cos\theta)^2}$

$\boxed{7.25}$ Two uniform rods joined experience an impulsive force

Find initial motion

(1) ______ $\hat{P}$ ______

7



$$V_{cm_1} = \frac{\hat{P}_0 + \hat{P}}{m}$$

$$\omega_1 = \frac{\frac{l}{2}(\hat{P}_0 + \hat{P})}{I} \qquad \frac{-3\hat{P}}{2ml}$$

$$V_{13} = V_{cm_1} - \frac{l}{2}\omega_1$$

(2)

$$\vec{V}_{13} = -V_{cm_2} - V_{13} \qquad = -\hat{P}/m$$

$$\vec{V}_{cm_2} = \frac{\hat{P}_0}{m} \qquad -\hat{P}/4m$$

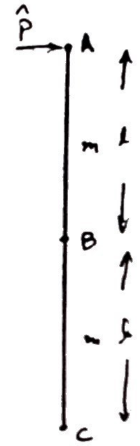
$$\vec{\omega}_2 = \frac{\hat{P}_0 \frac{l}{2}}{I} \qquad \frac{-9\hat{P}}{2ml}$$

vertical or horizontal

Now we solve these for variables!

method OK but unfinished

please see the key

7.25) 

I_{cm} for each rod $= \frac{1}{12} m\ell^2$

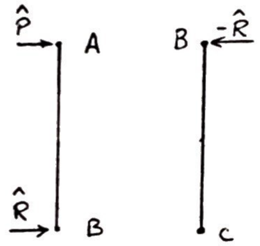
Let $\vec{v}_{c1}$ = initial velocity of cm of rod AB,

$\vec{v}_{c2}$ = " " " " " " BC,

ω_1 = " angular velocity of rod AB about its cm,

ω_2 = " " " " " BC " " "

Positive direction of $\vec{v}_{ci}$ is to right, of ω_i is out of paper.

Isolate rods: 

$\hat{R}$ represents impulse of rod AB on rod BC and the reaction impulse.

Conservation of momentum: $m\vec{v}_{c1} = \hat{P} + \hat{R}$, $\quad m\vec{v}_{c2} = -\hat{R}$

conservation of angular momentum: $\dfrac{(-\hat{P} + \hat{R})\ell}{2} = \dfrac{m\ell^2}{12}\omega_1$, $\quad \dfrac{\hat{R}\ell}{2} = \dfrac{m\ell^2}{12}\omega_2$.

Since the rods are joined at B,

$$v_B = v_{c1} + \frac{\ell}{2}\omega_1 = v_{c2} - \frac{\ell}{2}\omega_2 .$$

$$\Rightarrow \underbrace{\frac{\hat{P}+\hat{R}}{\cancel{m}} + \frac{3(\hat{R}-\hat{P})}{\cancel{m}}}_{-2\hat{P} + 4\hat{R}} = -\frac{\hat{R}}{\cancel{m}} - \frac{3\hat{R}}{\cancel{m}} = -4\hat{R}$$

$$\Rightarrow \underline{\hat{R} = \hat{P}/4}$$

$$\Rightarrow \boxed{\begin{aligned} &\vec{v}_{c1} = \frac{5}{4}\hat{P}/m, \quad \vec{v}_{c2} = -\hat{P}/4m \\ &\omega_1 = -\frac{9\hat{P}}{2m\ell}, \quad \omega_2 = \frac{3\hat{P}}{2m\ell} \\ &\vec{v}_B = -\hat{P}/m \end{aligned}}$$