# Ch 6: Many Particle Systems

6.1 Consider

$\begin{cases} r_1 = (1,1,1) \\ r_2 = (1,0,1) \\ r_3 = (0,0,1) \end{cases}$ $\begin{cases} v_1 = (-1,0,0) \text{ @ } m \\ v_2 = (0,2,0) \text{ @ } m \\ v_3 = (1,1,1) \text{ @ } m \end{cases}$ 3 particles positions & speeds

(a) $\vec{r}_{cm} = \dfrac{m\vec{r}_1 + m\vec{r}_2 + m\vec{r}_3}{m+m+m} = \frac{1}{3}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3)$

• $\vec{r}$ of CM: $= \frac{1}{3}((1,1,1)+(1,0,1)+(0,0,1))$

$\therefore \underline{\underline{\vec{r}_{cm} = \frac{1}{3}(2,1,3)}}$ ✓

(b) $\vec{v}_{cm} = \frac{1}{m_T}\sum m_i \vec{v}_i = \frac{m}{3m}((-1,0,0)+(0,2,0)+(1,1,1))$

• $\vec{V}$ of CM: $\underline{\underline{\vec{v}_{cm} = \frac{1}{3}(0,3,1)}}$ ✓

(c) the linear momentum of the system

• $\vec{p}$ of CM: $\underline{\underline{\vec{P}}} = m_T \vec{v}_{cm} = 3m\left(\frac{1}{3}(0,3,1)\right) = \underline{\underline{m(0,3,1)}}$ ✓

(d) angular mom. about origin

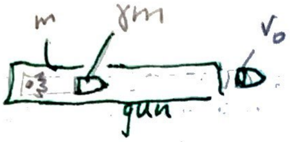
• $\vec{L}$ of System: $\vec{L} = \sum \vec{r}_i \times m_i \vec{v}_i = m\left[\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 0 & 2 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}\right]$

$m[(0,-1,1)+(-2,0,2)+(-1,1,0)] = \underline{\underline{m(-3,0,3)}}$ ✓

(e) the kinetic energy

KE: $\underline{\underline{T}} = \frac{1}{2}\sum m_i(\vec{v}_i \cdot \vec{v}_i) = \frac{m}{2}\sum(v_{x_i}^2 + v_{y_i}^2 + v_{z_i}^2) = \frac{m}{2}((1)+(4)+(3)) = \underline{\underline{4m}} = 4$

6.2

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what is the recoil speed of the gun.

$\gamma m v_0 = m v_g$ $\qquad \boxed{v_g = \gamma v_0}$ ✓

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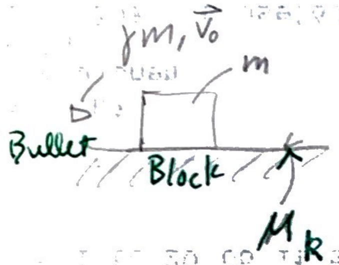
6.3

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Q: What fraction of original KE is lost as heat, upon impact?

Q: how far will it slide before coming to rest?

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$T: \; \frac{1}{2}(\gamma m)v_0^2 + 0 = \frac{1}{2} m v_f^2 + \frac{1}{2} m\gamma v_f^2 \Rightarrow \frac{m}{2}\gamma v_0^2 = v_f^2(1+\gamma)\frac{m}{2}$ $\qquad \boxed{v_f^2 = v_0^2 \frac{\gamma}{1+\gamma}}$

$\eta = 1 - \frac{v_f^2}{v_i^2} = 1 - \frac{v_0^2 \frac{\gamma^2}{(1+\gamma)^2}}{v_0^2} = 1 - \frac{\gamma}{1+\gamma}$ ✓ $= \boxed{\frac{1}{(1+\gamma)} \eta}$ No ✗

$\frac{1}{1+\gamma}$

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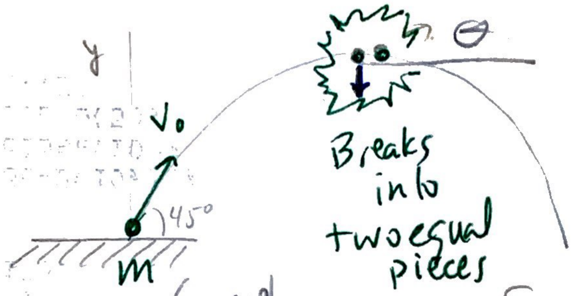
- work to slide $= F_f \cdot d = \mu(m+m\gamma)g\,d$
- work available $= KE(1-\eta)$ $\qquad \frac{\gamma m v_0^2}{2}(1-\eta) = \mu m(1+\gamma) g\,d$

$$\boxed{d = \frac{\gamma^2 v_0^2}{2\mu(1+\gamma)^2 g}}$$ ✓

6.4




- $1^{st}$ piece straight down @ $\sqrt{2}\,v_0$

Q: what is the direction and speed of the second particle?

$$(\cos 45°)v_0 = v_x = \frac{\sqrt{2}}{2} v_0$$

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At blast

$$\vec{p}_i = m v_x \hat{i}$$

what we want to find.

$$\vec{p}_f = \frac{m}{2}(\sqrt{2}\,v_0 \hat{j} + 0\,\hat{i}) + \frac{m}{2}(v_x' \hat{i} + v_y' \hat{j})$$

$$\vec{P}_i = \vec{P}_f = \frac{\cancel{m}}{2}\left((\sqrt{2}\,v_0 + v_y')\hat{j} + (0 + v_x')\hat{i}\right) = \cancel{m}(v_x \hat{i}) + 0\hat{j}$$

break into components;

$$\hat{j}: \quad v_0\frac{-\sqrt{2}}{2} + \frac{v_y'}{2} = 0 \qquad \boxed{v_y' = +v_0\sqrt{2}}$$

$$\hat{i}: \quad \frac{v_x'}{2} = v_x \qquad \boxed{v_x' = 2v_x = \sqrt{2}\,v_0}$$

$$\underline{\underline{v'}} = v_0\sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = \underline{\underline{2v_0}}$$

direction $\tan\theta = \frac{v_y}{v_x} = 1$

$$= \boxed{\theta = 45°}$$

$\boxed{6.6}$

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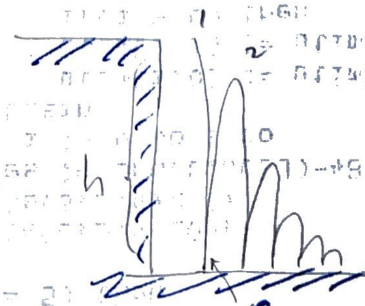


Show total verticle distance befor rebounds cease is

$$h\left(\frac{1+e^2}{1-e^2}\right)$$

$\epsilon$, coefficient of restitution

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(a) total vertical dist.

$$\epsilon = \frac{V_{\cancel{\text{}}}}{V_{\cancel{\text{}}}} = \frac{V_1}{V_2} \qquad V_1 \epsilon = V_2$$

1. $PE = KE \qquad mgh_1 = \frac{1}{2} m v_1^2$

$$\underline{v_1^2 = 2gh} \qquad \boxed{h_1 = \frac{v_1^2}{2g}}$$

2. $\frac{1}{2} m v_{g_{h_2}}^2 = mgh_2 \qquad \boxed{h_2 = \frac{V_2^2}{2g}} = \frac{v_1^2 \epsilon^2}{2g}$

3. $\frac{1}{2} m v_3^2 = mgh_3 \qquad h_3 = \frac{v_3^2}{2g} \quad (\epsilon^2 v_2)^2 \frac{1}{2g} = \frac{v_1^2 \epsilon^4}{2g}$

$$\boxed{h_T = \frac{V_1^2}{2g} + \frac{\epsilon^2 2}{\epsilon^2} h_1 \frac{V_1^2}{2g} + \frac{\epsilon^2 2}{\epsilon} h_1 \frac{V_1^2}{2g} + \cdots}$$

$$\frac{h_0}{1-\epsilon^2} \qquad \boxed{h_1 = \frac{2V_1^2}{2g} \sum_0^\infty \epsilon^{2n}_{2} \cdot h_0} = 1 + \frac{2h_0}{1-\epsilon^2} = \boxed{h\left(\frac{1+\epsilon^2}{1-\epsilon^2}\right) = h_T}$$

$$h_T = h_0\left(\frac{2}{1-\epsilon^2} - 1\right) = h_0\left(\frac{1-1+\epsilon}{1-\epsilon^2}\right) \quad 1-\epsilon^2+2$$

finally

$$\boxed{\sum_0^\infty x^n = \frac{1}{1-x} - h_0} \qquad \text{let } x = \epsilon^{-2}$$

so

$$= \sum_{n=1}^{\infty} \epsilon^{+2n} = \frac{1}{1-\frac{1}{\epsilon^2}}$$

6.6 cont. (b) Total time elapsed

$mgh = \frac{1}{2} m v^2$

$h = \frac{v^2}{g}$

$h_0 = \frac{1}{2} g t_1^2 \; ; \; \left(\frac{2h_0}{g}\right)^{\frac{1}{2}} = t_1$

$v_2 = \epsilon v_1$

$h_1 = \frac{v_2^2}{g} = \frac{1}{2} g t_2^2 \; ; \; 2\left(\frac{2v_2^2}{g^2}\right)^{\frac{1}{2}} = t_2 = 2\left(\frac{2\epsilon^2 v_1^2}{g^2}\right)^{\frac{1}{2}}$

$h_2 = \frac{v_3^2}{g} = \frac{1}{2} g t_3^2 \; ; \; 2\left(\frac{2v_3^2}{g^2}\right)^{\frac{1}{2}} = t_3 = 2\left(\frac{2\epsilon^4 v_1^2}{g^2}\right)^{\frac{1}{2}}$

$t_T = t_1 + t_2 + t_3 \ldots$

$= \left(\frac{2h_0}{g}\right)^{\frac{1}{2}} + 2\left(\frac{2\epsilon^2 h_0}{g}\right)^{\frac{1}{2}} + 2\left(\frac{2\epsilon^4 h_0}{g}\right)^{\frac{1}{2}} + \ldots$

$t_T = \left(\frac{2h_0}{g}\right)^{\frac{1}{2}} + 2\sum_{n=1}^{\infty}\left(\frac{2h_0}{g}\epsilon^{2n}\right)^{\frac{1}{2}}$ let $\left(\frac{2h_0}{g}\right)^{\frac{1}{2}} = L$

$t_T = L + 2\sum L\epsilon^n = L\left(1 + 2\sum\epsilon^{2n}\right) = L\left(1 + \frac{2}{1-\epsilon^2}\right)$

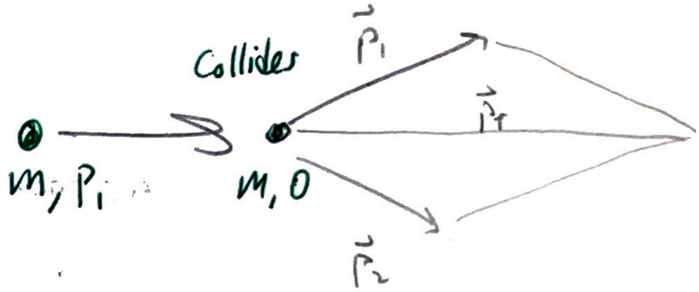
$$\boxed{t_T = \sqrt{\frac{2h_0}{g}}\left(\frac{3-\epsilon^2}{1-\epsilon^2}\right)}$$

$\sqrt{\frac{2h}{g}} \; \frac{1+\epsilon}{1-\epsilon}$

please see the key

$\boxed{6.17}$

$$\frac{(\vec{p}_1)^2}{2m} = \frac{(\vec{p}_1')^2 + (p_2')^2}{2m}$$




Find Total energy loss, Q

$$\vec{p} \Rightarrow \vec{p}_1 = \vec{p}_1' + \vec{p}_2'$$

$$E \Rightarrow \frac{p_1^2}{2m} = \frac{p_1'^2}{2m} + \frac{p_2'^2}{2m} + Q$$

$$p_1^2 = p_1'^2 + p_2'^2 + \underline{2mQ}$$

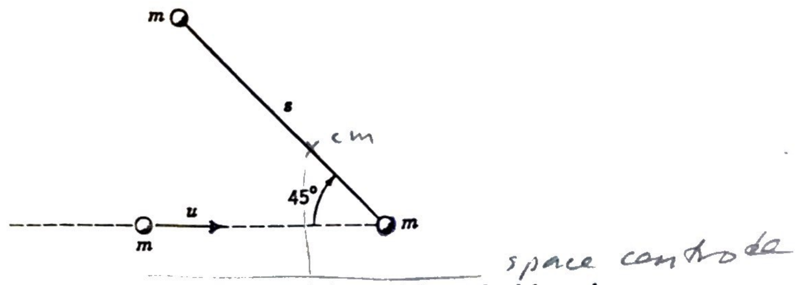
law of cosines $\Rightarrow -2p_1'p_2'\cos\psi$

$$Q = \frac{p_1' p_2' \cos\psi}{m}$$

✓

Problem from Bradbury:

8.14.34 Two particles of mass $m$ are at rest and connected by a rigid massless rod of length $s$. A third particle also of mass $m$, and moving with a velocity $u$




at an angle of 45° with the rigid rod, strikes one of the particles and sticks to it as in the figure. Find the linear velocity of the center of mass and the angular velocity of rotation of the system after the collision. Locate the space centrode.

Conservation of linear momentum:

$$mu = 3m v_c \quad \Rightarrow \quad v_c = \frac{1}{3} u$$

CM is $\frac{1}{3}s$ up from lower mass after third particle sticks to rigid body.

Conservation of angular momentum about this point (which is CM right after collision):

$$m u \underbrace{\frac{s}{3\sqrt{2}}}_{\text{vertical distance from line of } u \text{ to CM.}} = I_c \omega = m\left[2\left(\frac{s}{3}\right)^2 + \left(\frac{2s}{3}\right)^2\right]\omega = \frac{2}{3} m s^2 \omega$$

$$\Rightarrow \omega = \frac{u}{2\sqrt{2}\, s}$$

Space centrode: $b\omega = v_c$

$$\Rightarrow b = \frac{2\sqrt{2}}{3} s$$

Roy W. Erickson

PHYSICS 321 TAKE-HOME QUIZ NOV. 13, 1981

Due: 1:10 p.m., Nov. 16

Open Book - you may use your text, notes, and problem solutions. You may also use math tables. You may not use any other books or consult with anyone except Dr. Evenson.

1. A proton of mass $m_p$ with initial velocity $v_o$ collides with a helium atom, mass $4m_p$, that is initially at rest. If the proton leaves the point of impact at an angle of 45° with its original line of motion, find the final velocities of each particle.

   Assume that the collision is inelastic and that Q is equal to 1/4 of the initial energy of the proton.

   Solve the problem in the lab frame. Start from first principles.

see attached 1st, then back

In c.m. frame

$$\underline{\underline{\bar{V}_1}} = \frac{m_2 \vec{v}_1}{m_1+m_2} \Rightarrow \frac{4m_p v_0}{m_p+4m_p} = \underline{\underline{\frac{4}{5}v_0}}$$

$$\underline{\underline{\gamma}} = \frac{m_p}{4m_p}\left[1-\frac{1}{4}\left(1+\frac{m_p}{4m_p}\right)\right]^{-\frac{1}{2}} = \frac{1}{4}\left[1-\frac{1}{4}\left(\frac{5}{4}\right)\right]^{-\frac{1}{2}} = \frac{\sqrt{16}}{4\sqrt{11}} \rightarrow \underline{\underline{\tfrac{1}{11}}}$$

From text p179 eq 6.47

$$\frac{v_1' \sin\varphi_1}{\sin\theta} = \bar{v}_1'$$

$$v_1'\left(\frac{\sin 45}{\sin 32.690}\right) = \bar{v}_1'$$

$$v_1'(.5401) = \bar{v}_1' \qquad (4)$$

$$\tan\varphi_1 = \frac{\sin\theta}{\gamma+\cos\theta} = \tan 45^0 = 1$$

—or—

$$\sin\theta = \gamma + \cos\theta$$

$$\{\sin\theta - \cos\theta = \gamma\}^2$$

$$s^2\theta - 2s\theta c\theta + c^2\theta = \gamma^2$$

$$1-\sin 2\theta = \gamma^2$$

$$\frac{\sin^{-1} 1-\gamma^2}{2} = \theta$$

$$\underline{\underline{\theta}} = \frac{\sin^{-1}\left(1-\frac{1}{11}\right)}{2} = \underline{\underline{32.690}}$$

From text p179 eq. 6.42

$$\frac{\bar{P}_1^2}{2m} = \frac{\bar{P}_1'^2}{2m} + Q$$

$$M = \frac{m_1 m_2}{m_1+m_2} = \frac{4}{5}m_p$$

$$\frac{m_p\left(\frac{4}{5}v_0\right)^2}{2\cdot\frac{4}{5}m_p} = \frac{m_p \bar{v}_1'^2}{2\cdot\frac{4}{5}m_p} + \left\{Q = \frac{1}{4}\left(\frac{1}{2}m_p v_0^2\right)\right\}$$

$$\frac{\frac{16}{25}v_0^2}{8/5} = \frac{5}{8}\bar{v}_1'^2 + \frac{1}{8}v_0^2$$

$$\frac{2}{5}v_0^2 - \frac{5}{8}\bar{v}_1'^2 \stackrel{?}{=}$$

use (4)

$$\frac{2}{5}v_0^2 - \frac{5}{8}v_1'^2(.5401)^2 =$$

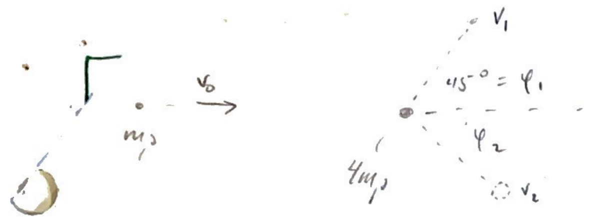
Test This eq for Both $v_1'$ → $+] v_1' \Rightarrow c = .42$, $-] v_1' \Rightarrow c = .84$

$$+] \quad \frac{2}{5} - \frac{5}{8}(.421)^2(.5409)^2 = .37$$

$$-] \quad \frac{2}{5} - \frac{5}{8}(8405)^2(.5401)^2 = .27 \leftarrow \text{close}$$

well I guess the "−" case.

$$\boxed{\begin{aligned} V_1 &= .84 v_0 \\ V_2 &= .104\ v_0 \end{aligned}}$$




What are the final velocities of each particle

Two equations : $KE_i = KE_f + Q$

$$\frac{1}{2} m_p v_0^2 = \frac{1}{2} m_p v_1'^2 + \frac{1}{2} 4m_p v_2'^2 + \frac{1}{4}\left(\frac{1}{2} m_p v_0^2\right)$$ ✓

or

$$\boxed{\frac{3v_0^2}{4} = v_1'^2 + 4v_2'^2} \qquad (1)$$

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: $\vec{p}_i = \vec{p}_f$

$$m_p \vec{v}_0 = m_p \vec{v}_1' + 4m_p \vec{v}_2'$$

or

$$v_0\begin{pmatrix}1\\0\end{pmatrix} = v_1'\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} + 4v_2'\begin{pmatrix}\cos\varphi_2\\ -\sin\varphi_2\end{pmatrix}$$

separates to

$$\boxed{\begin{aligned} v_0 &= \frac{v_1}{\sqrt{2}} + 4v_2\cos\varphi_2 \qquad (2)\\ 0 &= \frac{v_1}{\sqrt{2}} - 4v_2\sin\varphi_2 \qquad (3)\end{aligned}}$$ ✓

three equations, three unknowns:

eq. 3 $\Rightarrow$ $\sin\varphi_2 = \frac{v_1}{4\sqrt{2}\,v_2'}$

eq 2 + (- eq 3) $\Rightarrow$

$$v_0 = 4v_2'(\cos\varphi_2 + \sin\varphi_2)$$

$$v_0^2 = 16v_2^2(1 + 2\cos\varphi_2\sin\varphi_2)$$

$$= 16v_2^2\left(1 + 2\sqrt{1 - \left(\frac{v_1}{4\sqrt{2}v_2}\right)^2}\left(\frac{v_1}{4\sqrt{2}v_2}\right)\right)$$

$$= 16v_2^2 + \left(\frac{16^2 v_2^4 \cdot 4 \cdot v_1^2}{16\cdot 2 v_2^2} - \frac{2\cdot v_1^2\, 16^2 v_2^4\, v_1^2}{16\cdot 2v_2^2\; 16\cdot 2v_2^2}\right)^{1/2}$$

$$(v_0^2 - 16v_2^2)^2 = \left[32v_2^2 v_1^2 - v_1^4\right]$$

$$v_0^4 - 2\cdot 16v_2^2 v_0^2 + 16^2 v_2^4 - 32v_2^2 v_1^2 + v_1^4 = 0$$

eq 1 $\Rightarrow$ $v_1^2 = \frac{3}{4}v_0^2 - 4v_2^2$

$$\therefore \quad v_0^4 - 32 v_2^2 v_0^2 + 16^2 v_2^4 - 32 v_2^2\left[\frac{3}{4}v_0^2 - 4v_2^2\right] + \left[\frac{3}{4}v_0^2 - 4v_2^2\right]^2 = 0$$

$$v_0^4 - 32 v_2^2 v_0^2 + 16^2 v_2^4 - \frac{32\cdot 3 v_2^2 v_0^2}{4} + 4\cdot 32 v_2^4 + \left[\frac{9}{16}v_0^4 - \frac{2\cdot 3\cdot 4}{4} v_0^2 v_2^2 + 16 v_2^4\right] = 0$$

$$v_0^4\left[1 + \frac{9}{16}\right] + v_0^2 v_2^2\left[-32 - 24 - 6\right] + v_2^4\left[16^2 + 128 + 16\right] = 0$$

$$v_0^4\left[16 + 9\right] + v_0^2 v_2^2\left[-16\cdot(62)\right] + v_2^4\left[(16)(400)\right] = 0$$

$$v_0^4\, 10 - v_0^2 v_2^2\, 992 + v_2^4\, 6400 = 0$$

$$v_0^4 - v_0^2 v_2^2\, 99.2 + v_2^4\, 640 = 0$$

$a = 640$
$b = -99.2$
$c = 1$

$$v_2^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$v_2^2 = \frac{99.2 v_0^2 \pm \left((99.2)^2 - 4(640)(1)\right)^{1/2} v_0^2}{2(640)}$$

$$v_2^2 = \frac{99.2}{1280} v_0^2 \pm \frac{85.33}{1280} v_0^2$$

$$v_2^2 = (.0775 \pm .06661)\, v_0^2$$

$+$: $\quad \underline{v_2 = .379\, v_0}$

$-$: $\quad \underline{v_2 = .1044\, v_0}$

Total KE

from eq(1):

$$v_1 = \sqrt{\frac{3}{4}v_0^2 - 4v_2^2}$$

$$+]\ v_1 = \sqrt{\frac{3}{4} - 4(.379)^2}\ v_0 = .42\, v_0$$

$.41885$

$$-]\ v_1 : \left(\frac{3}{4} - 4(.1044)^2\right)^{\frac{1}{2}} v_0 = .84\, v_0$$

SEE BACK OF PROBLEM SHEET.

In

From text

$v_1' \frac{\sin \varphi}{\sin \theta}$

$v_1'\left(\frac{\sin 45}{\sin 32}\right.$

$v_1'(.540$

From text

$\frac{\bar{P}_1^2}{2m}$

$\mu =$

Test This eq for Both $v_1'$

6.18

Rocket fired upward

a) Find eq. of motion —

b) Find ratio $\left(\frac{m_{fuel}}{m_{payload}}\right)$ if $v_f = v_e$ ; $V = kv_e$ , $\dot{m} = \dot{m}$

c) Find numerical ratio if $k = \frac{1}{4}$ ; $\frac{m_{f_0}}{\dot{m}} = 100$

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(a) For variable mass

$$\vec{F} = m\dot{\vec{v}} + \vec{V}\dot{m} \Rightarrow -mg = m\ddot{z} + V\dot{m}$$

$$\boxed{\ddot{z} = -g - \frac{V\dot{m}}{m(t)}}$$

(b) $d\dot{z} = \left(-g - V\frac{\dot{m}}{m}\right)dt$

$= -gdt - V\frac{dm}{m}$ or $\int_0^v dv = -gt - V\ln m\Big|_{m_0}^{m} = -gt - V\frac{m(t)}{m_0}$

so $v(t) = -gt - kv_e \ln\frac{m_{r+f}(t)}{m_{r+f}(0)}$

$m(t) = m_0 + \dot{m}t$ when all fuel is gone $m(t_f) = m_{payload}$

$m_p = m_0 + \dot{m}t_f \rightarrow t_f = \frac{m_p - m_0}{\dot{m}}$

$V(t_f) \equiv v_e$ $\therefore$ $v_e = -g\left(\frac{m_p - m_0}{\dot{m}}\right) - kv_e \ln\frac{m_p}{m_0}$

which is written as $e^{\left[\frac{1}{k} + \frac{gm_f}{\dot{m}kv_e}\right]} = \frac{m_p + m_f}{m_p} = 1 + \frac{m_f}{m_p}$

therefore $\boxed{\frac{m_f}{m_p} = e^{\left(\frac{1}{k} + \frac{gm_f}{kv_e\dot{m}}\right)} - 1}$ ✓

(c) $\frac{m_f}{m_p} = e^{\left(4 + 400s\left(\frac{9.8\,m/s^2}{1.1\times10^4\,m/s}\right)\right)} - 1 = \underline{53.8}$

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