Celestial Mechanics

5.1 Derive Kepler's Law $F_g = F_c$

5/5

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \qquad \text{but } v = \omega r$$

$$\frac{GM}{r} = \omega^2 r^2 \qquad GM = \omega^2 r^3 = \left(\frac{2\pi}{T}\right)^2$$

or $\boxed{T = \frac{2\pi}{\sqrt{GM}}\, r^{3/2}}$

5.4 (a) verify $\vec{F}$ inside a thin crusted hollow earth

TOTAL

TOP view

Show $g=0$ from crust A - crust B

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A mass of shell $= \rho 4\pi r^2$

$ds' = b\,d\theta \qquad ds = a\,d\theta$

$\boxed{dA' = b^2\, d\theta\, d\phi \sin\theta} \qquad \boxed{dA = a^2\, d\theta\, d\phi \sin\theta}$

$dm' = \rho b^2\, d\theta\, d\phi \qquad dm = \rho a^2\, d\theta\, d\phi$

$dg' = G\frac{dm'}{b^2}$ or $dg = G\frac{dm}{a^2}$

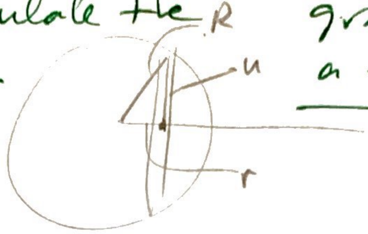
$$g' = G\iint \frac{\rho b^2\, d\theta\, d\phi \sin\theta}{b^2} = G\rho 4\pi$$

$$g = G\iint \frac{\rho a^2\, d\theta\, d\phi \sin\theta}{a^2} = -G4\pi\rho$$

$$g' + g = G\rho 4\pi - G4\pi\rho = 0$$

so $\boxed{F = mg = 0}$

5.4 (b) Calculate the gravitational potential Φ insi a thin shelled hollow earth



$$\Phi = -G \int \frac{dm}{u} \qquad r^2 + u^2 = R^2$$

$$= -G \int \frac{2\pi \rho R^2 \sin\theta \, d\theta}{(R^2 - r^2)^{1/2}} \qquad R\cos\theta = r$$

so

$$\Phi = -2G \int_0^{\pi/2} \frac{(2\pi\rho R^2)(\sin\theta \, d\theta)}{R(1-\cos^2\theta)^{1/2}} = -2G\,2\pi\rho R \int_0^{\pi/2} d\theta$$

$$\boxed{\Phi = -2G\pi^2 \rho R = \text{const}}$$

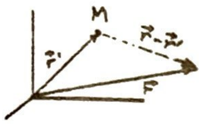
so $\vec{F} = \vec{\nabla}\Phi = 0$ ✓

CALCULATING GRAVITATIONAL FIELDS AND POTENTIALS BY DIRECT INTEGRATION

The field at $\vec{r}$ due to a point mass M at $\vec{r}\,'$ is just

$$\boxed{\vec{g}(\vec{r}) = -\frac{GM\,(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}}$$

(Newton's law of universal gravitation)



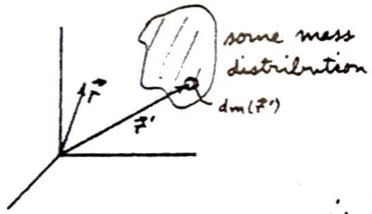
I.e. $\vec{g}(\vec{r})$ is in the direction of a line from $\vec{r}$ to M and varies inversely as the square of the distance from M to $\vec{r}$ (i.e. $|\vec{r}-\vec{r}\,'|$).

The point where M is located, $\vec{r}\,'$, is called the source point. The point where the field is to be calculated, $\vec{r}$, is called the field point.

Similarly, the potential in the same situation is (taking $\psi(\infty) = 0$)

$$\psi(\vec{r}) = \frac{-GM}{|\vec{r}-\vec{r}\,'|}.$$

For any continuous distribution of mass, we must add up vectorially the contributions to $\vec{g}(\vec{r})$ due to infinitesimal elements of mass at $\vec{r}\,'$, letting $\vec{r}\,'$ vary through the distribution of mass. E.g.



$d\vec{g}(\vec{r})$ = contribution to $\vec{g}(\vec{r})$ due to element of mass $dm(\vec{r}\,')$ at $\vec{r}\,'$.

$$= -\frac{G\,dm(\vec{r}\,')\,(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}.$$

$$\therefore\ \vec{g}(\vec{r}) = -G\int \frac{dm(\vec{r}\,')\,(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}.$$

What is $dm(\vec{r}\,')$?

For a volume distribution of mass, $\rho(\vec{r}\,')$: $dm(\vec{r}\,') = \rho(\vec{r}\,')\,d^3r' = \rho(\vec{r}\,')\underbrace{d\Sigma'}_{\text{volume element}}$

For a surface distribution of mass, $\sigma(\vec{r}\,')$: $dm(\vec{r}\,') = \sigma(\vec{r}\,')\underbrace{d\sigma'}_{\text{area element}}$

For a line distribution of mass, $\mu(\vec{r}\,')$: $dm(\vec{r}\,') = \mu(\vec{r}\,')\underbrace{ds'}_{\text{line element}}$

Similarly, for $\psi(\vec{r})$: $\boxed{\psi(\vec{r}) = -G\int \frac{dm(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}}$.

REMEMBER:

Make use of symmetry in the charge distribution when calculating $\vec{g}(\vec{r})$.
Be careful to properly include the coordinate dependence of unit vectors for non-rectangular coordinates in the integrals.

$$|\vec{r}-\vec{r}\,'| = \sqrt{(\vec{r}-\vec{r}\,')\cdot(\vec{r}-\vec{r}\,')} = \sqrt{r^2 + r'^2 - 2\,\vec{r}\cdot\vec{r}\,'}.$$

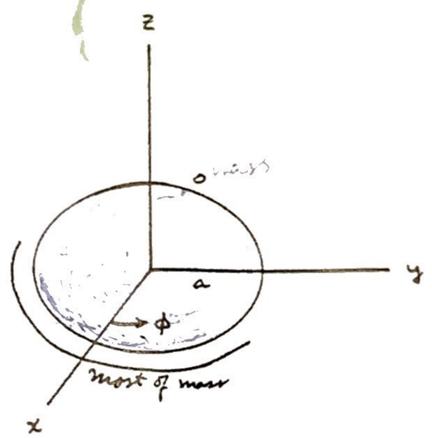
Q: how do you use this for prob 5.4a

A Fairly Difficult Example

Consider a disk of radius a in the xy-plane, centered at the origin. It has a surface mass density

$$\sigma(\vec{r}) = \frac{Mr}{a^3}\left|\cos\frac{\phi}{2}\right| \quad \text{(in cylindrical coordinates)}$$

Find the gravitational field on the axis of the disk, above the disk, by direct integration.



$$\vec{r} = z\hat{k}, \qquad \vec{r}\,' = r'\cos\phi'\,\hat{\imath} + r'\sin\phi'\,\hat{\jmath}, \qquad |\vec{r}-\vec{r}\,'| = \sqrt{r'^2+z^2}.$$

$$d\vec{g}(\vec{r}) = -\frac{GMr'}{a^3}\left|\cos\frac{\phi'}{2}\right| dr'\,r'\,d\phi'\,\frac{(z\hat{k} - r'\cos\phi'\,\hat{\imath} - r'\sin\phi'\,\hat{\jmath})}{(r'^2+z^2)^{3/2}}.$$

Symmetry $\Rightarrow \vec{g}(z)$ *has no y-component.*

$$\Rightarrow \vec{g}(z) = -\frac{GM}{a^3} z\hat{k}\int_{-\pi}^{\pi} d\phi'\cos\frac{\phi'}{2}\int_0^a dr'\frac{r'^2}{(r'^2+z^2)^{3/2}} + \frac{GM}{a^3}\hat{\imath}\int_{-\pi}^{\pi} d\phi'\cos\frac{\phi'}{2}\cos\phi'\int_0^a dr'\frac{r'^3}{(r'^2+z^2)^{3/2}}$$

Let $u = r'^2+z^2$, $du = 2r'dr'$ $\Rightarrow$

$$\int_0^a dr'\frac{r'^3}{(r'^2+z^2)^{3/2}} = \frac{1}{2}\int_{z^2}^{a^2+z^2} du\,(u-z^2)\,u^{-3/2}$$

$$= \frac{1}{2}\left[2u^{1/2} + 2z^2u^{-1/2}\right]_{z^2}^{a^2+z^2} = \sqrt{a^2+z^2} - z + \frac{z^2}{\sqrt{a^2+z^2}} - z$$

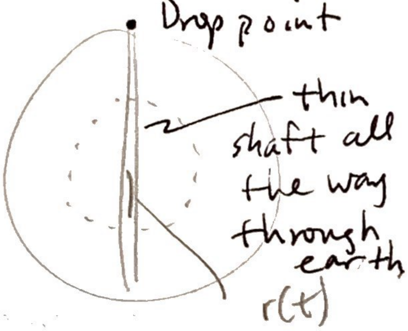
$$= \frac{2z^2+a^2}{\sqrt{z^2+a^2}} - 2z$$

$$\int_0^a dr'\frac{r'^2}{(r'^2+z^2)^{3/2}} = \left[-\frac{r'}{\sqrt{r'^2+z^2}} + \ln\left(r'+\sqrt{r'^2+z^2}\right)\right]_0^a = -\frac{a}{\sqrt{z^2+a^2}} + \ln\left(\frac{a+\sqrt{z^2+a^2}}{z}\right)$$

$$\int_{-\pi}^{\pi} d\phi'\cos\frac{\phi'}{2} = 2\sin\frac{\phi'}{2}\Big|_{-\pi}^{\pi} = 2+2 = 4. \qquad \int_{-\pi}^{\pi} d\phi'\cos\frac{\phi'}{2}\cos\phi' = \frac{4}{3}.$$

$$\Rightarrow \vec{g}(z) = -\frac{4GM}{a^3}\left[\frac{-az}{\sqrt{z^2+a^2}} + z\ln\left(\frac{a+\sqrt{z^2+a^2}}{z}\right)\right]\hat{k} + \frac{4GM}{3a^3}\left[\frac{2z^2+a^2}{\sqrt{z^2+a^2}} - 2z\right]\hat{\imath}. \qquad \underline{(z>0)}$$

$\boxed{5.5}$ Show a particle dropped into shaft will have S.H. Oscillation



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- We know from prob 5.4 that there is no force contribution from outside of $r(t)$.

- We also know that the mass inside $r(t)$ contributes a point-like force: $F(r) = -G\frac{M(r)\,m}{r^2}$

The eq. of motion

$$m\ddot{r} = F = -\frac{GM(r)\,m}{r^2} = -\frac{G\left(\frac{4}{3}\pi r^3 \rho\right)}{r^2} = -\frac{4}{3}\pi\rho G r m$$

density of earth

or $\boxed{\ddot{r} + \underbrace{\tfrac{4}{3}\pi\rho G}_{\omega^2}\, r = 0}$ ← SHM eq

so $\omega = \frac{2\pi}{T}$ $\quad T = \frac{2\pi}{\omega}$

$$\boxed{T = \cancel{2\pi}\sqrt{\frac{3\pi}{\cancel{4}\rho G}}}$$

excellent work!

$\boxed{5.2}$ Find the radius required for a synchronous satellite in circular orbit.

5/5

$$-\vec{F}_g = \vec{F}_{cent}$$

$$\frac{GMm}{r^2} = m\frac{V^2}{r} \quad \text{so} \quad V = \omega r \qquad \frac{GM}{r} = \omega^2 r^2$$

$$\frac{GM}{\left(\frac{2\pi}{T}\right)^2} = r^3 \qquad \underline{\underline{r}} = \sqrt[3]{\frac{GMT^2}{(2\pi)^2}} = \left(\frac{\left(6.7\times10^{-11}\,\frac{N\text{-}m^2}{kg^2}\right)\left(6\times10^{24}\,kg\right)\left(24\,hr\right)^2}{2^2\pi^2}\right)^{\frac{1}{3}}$$

$kg\,m/s^2$

$$= \left(\frac{(6.7)(6)(24\,hr)^2(3600\,s/hr)^2 \times 10^{-11} \times 10^{24}\,m^3}{4\pi^2}\right)^{1/3} = \underline{\underline{4.24\times10^7\,m}}$$

10^{7}

5.3 Compute mass of earth: $T_{moon} = 27.3$ days.

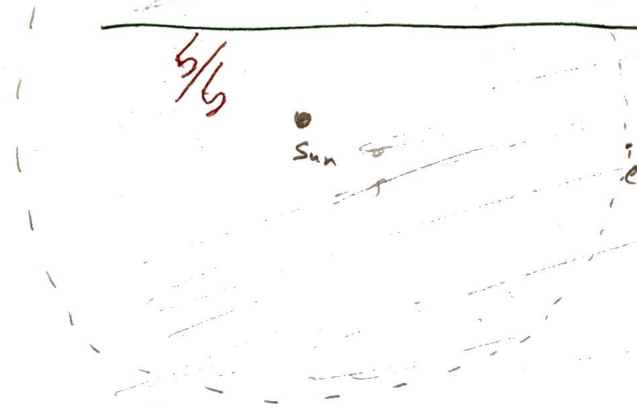
$R_m = 3.84\times10^5\,Km$.

5/5

$$\frac{R_m^3(2\pi)^2}{GT^2} = \underline{\underline{M_e}} = \frac{(3.84\times10^5\,Km)^3(2\pi)^2 \times \left(1000\,\frac{m}{Km}\right)^3}{\left(6.7\times10^{-11}\,\frac{m^3/s^2}{kg}\right)\left(27.3\,d\ 24\,\frac{hr}{d}\ 3600\,\frac{h}{s}\right)} = \underline{\underline{6.0\times10^{24}\,kg}}$$

$\boxed{5.7}$ If solar system embedded in uniform dust cloud density ρ.
If constant dust cloud what is force law on earth (or any planet w/ orbit "r" from Sun)?

5/5



The mass inside earth orbit sphere attracts earth.

$$m_{cloud} = \left(\frac{4}{3}\pi\rho R_e^3\right)$$

$$F = -G\left(M_s + M_c\right)\frac{m_e}{R_e^2} = -G\frac{M_s m_e}{R_e^2} - G\left(\frac{4}{3}\pi\rho R_e^3\right)\frac{m_e}{R_e^2}$$

$$\boxed{F = -G\frac{M_s m_e}{R_e^2} - \frac{4}{3}G\pi\rho\, m_e R_e}$$

$\boxed{5.8}$ if particle follows $r = r_0 e^{k\theta}$ show that force is inverse cube & θ varies log with t.

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$$\frac{d^2u}{d\theta^2} + u = -\frac{1}{mh^2u^2} f(u^{-1}) \qquad u = \frac{1}{r_0}e^{-k\theta}$$

$$\frac{du}{d\theta} = \frac{1}{r_0} - k e^{-k\theta}$$

$$-mh^2\left(\frac{1}{r_0^2}e^{-2k\theta}\right)\left(\left(\frac{k^2}{r_0}e^{-k\theta}\right) + \frac{1}{r_0}e^{-k\theta}\right) = f(r)$$

$$\frac{d^2u}{d\theta^2} = \frac{-k(-k)e^{-k\theta}}{r_0} = \frac{k^2}{r_0}e^{-k\theta}$$

so $$f(r) = -mh^2\left(\frac{k^2}{r_0^3 e^{3k\theta}} + \frac{1}{r_0^3 e^{3k\theta}}\right)$$

$$\boxed{f(r) = -mh^2\left(k^2 + 1\right)\frac{1}{r^3}}$$

$\boxed{\text{5.8 cont.}}$ show θ varies logrithmically w/ t.

$r = r_0 e^{k\theta}$

$$r^2\dot{\theta} = h$$

$$\int_{\theta_0}^{t} e^{2k\theta}\, d\theta = \int_{\theta}^{} \frac{h}{r_0^2}\, dt$$

$$\frac{e^{2k\theta}}{2k} + C = \frac{ht}{r_0^2}$$

$$\boxed{\theta = \frac{1}{2k}\ln\left(\frac{ht}{r_0^2} - c\right)2k}$$

close
eval limits

$\boxed{5.11}$ A particle moves in a spirial orbit $r = a\theta$

if $\theta = kt$ is the force a central field?

5/5

$r^2\dot{\theta} = h$ for a central field.

$u = \frac{1}{a\theta}$

$\frac{du}{d\theta} = -\frac{1}{a\theta^2}$

$\frac{d^2u}{d\theta^2} = +\frac{2}{a\theta^3}$

So

$$(a\theta)^2\dot{\theta} = h$$

$$\Rightarrow \quad \theta^2 d\theta = \frac{h}{a^2}\, dt$$

integrating

$$\frac{\theta^3}{3} + C = \frac{ht}{a^2}$$

$$\underline{\theta = \left(\frac{3ht}{a^2} + c\right)^{\frac{1}{3}}}$$

? θ should vairy by $t^{1/3}$ for it to be a central field. (not linearly as asked)

In a central field $\dot{\theta} \neq$ const because

$$\dot{\theta} = \frac{h}{r^2} = \frac{h}{a^2\theta^2} \neq \text{const}$$

therefore $\dot{\theta} = kt$ the force is not central

$\boxed{5.13}$ a. Compute the period of Halley's Comet
b. Find speed at perihelion. (closest to sun)
c. Find speed at aphelion.

$e = .967$ perihelion dist $= 5.5 \times 10^7$ miles

a.

$$T = c\,a^{3/2} \qquad c = 2\pi (GM)^{-\frac{1}{2}} \qquad r_1 = r_0 \frac{1+e}{1-e}$$

$$2a = r_1 + r_0$$

$$a = \frac{r_0}{2}\left(1 + \frac{1+e}{1-e}\right)$$

so $$T = \frac{2\pi}{\sqrt{GM}}\left[r_0 \frac{1}{(1-e)}\right]^{3/2} \qquad a = r_0\left(\frac{1}{1-e}\right)$$

but if r_0 is in a.u. & T years then
c = unity.

so $$T = \left(1 \frac{yr}{a.u.}\right)\left(\frac{5.5 \times 10^7 \text{ miles}}{9.3 \times 10^7 \text{ miles}/a.u.}\right)^{3/2}\left(\frac{1}{1-.967}\right)^{3/2}$$

$$\boxed{T = 75.87 \text{ yrs}}$$

b. Knowing $e = (V_0 / V_c)^2 - 1$

V_{circle} comes from $\frac{\cancel{m} V_0^2}{\cancel{r_0}} = \frac{GM\cancel{m}}{r_0^{\cancel{2}}}$

$$V_0 = \sqrt{e+1}\,V_c$$

$$V_0 = \left(\sqrt{.967+1}\right)\left(\sqrt{GM/r_0}\right) = \sqrt{\frac{(.967+1)\left(6.67 \times 10^{-11} \frac{Nt\text{-}m^2}{kg^2}\right)\left(2 \times 10^{30} \text{ kg}\right)}{\left(5.5 \times 10^7 \text{ miles}\right)\left(1609 \frac{m}{mile}\right)}}$$

$$= 5.5 \times 10^4 \text{ meters/sec}$$

$$\boxed{V_0 = 12 \times 10^4 \text{ mi/hr.}}$$

c.

$$h = r^2\dot\theta = r_0^2\dot\theta_0 = r_0 V_0 = r_1 V_1 \qquad V_1 = \frac{12 \times 10^4 \text{ mi/hr}}{\left(\frac{1+.967}{1-.967}\right)} = 2 \times 10^3 \text{ mi/hr} = V_1$$

$\boxed{5.19}$ Show that $m\ddot{r} - \frac{mh^2}{r^3} = f(r)$, is the same as that of a part. in rectilinear motion in an effective potential $U(r)$; $U(r) = V(r) + \frac{1}{2}\frac{mh^2}{r^2}$. Here $f(r) = -\frac{dV(r)}{dr}$, make a rough plot of $U(r)$ for $V(r) = \frac{-k}{r}$ (stable) & $V(r) = \frac{-k}{r^3}$ (unstable).

For rectilinear motion also

$$E = \frac{1}{2}mv^2 + V(r) \qquad v^2 = \langle \vec{v}, \vec{v} \rangle = \langle \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta ; \dot{r}, r\dot{\theta} \rangle$$

therefore

$$E = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + V(r) \qquad \text{but } r^2\dot{\theta} = h$$

$$= \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\left(\frac{h^2}{r^4}\right) + V(r)$$

$$= \frac{1}{2}m\dot{r}^2 + \left[\frac{1}{2}m\frac{h^2}{r^2} + V(r)\right]$$

since this component is of position only, it can be included in the "effective" potential

So

$$\underline{E = \frac{1}{2}m\dot{r}^2 + V_{eff}(r)} \qquad \underline{V_{eff}(r) = V(r) + \frac{1}{2}m\frac{h^2}{r^2}}$$

$f(r)$ & h come from:

$$\underline{\underline{f(r)}} = ma_r = m(\ddot{r} - r\dot{\theta}^2) = m\ddot{r} - rm\frac{h^2}{r^4} = \underline{\underline{m\ddot{r} - \frac{mh^2}{r^3}}}$$

$$0 = ma_\theta = m(\underbrace{2\dot{r}\dot{\theta} + r\ddot{\theta}}_{=0}) = m\frac{d(r^2\dot{\theta})}{dt}$$

(see back) so $\boxed{r^2\dot{\theta} = \text{const} \equiv h}$

another way (the way they ask, I think)

$$m\ddot{r} - \frac{mh^2}{r^3} = f(r)$$

so

$$\boxed{\text{let } m\ddot{r} = F(r) \quad \& \quad \frac{\partial U}{\partial r} = -F(r)}$$

it become

$$-\frac{\partial U}{\partial r} - \frac{mh^2}{r^3} = -\frac{\partial V}{\partial r}$$

integrate

$$-U + \frac{mh^2}{2r^2} = -V$$

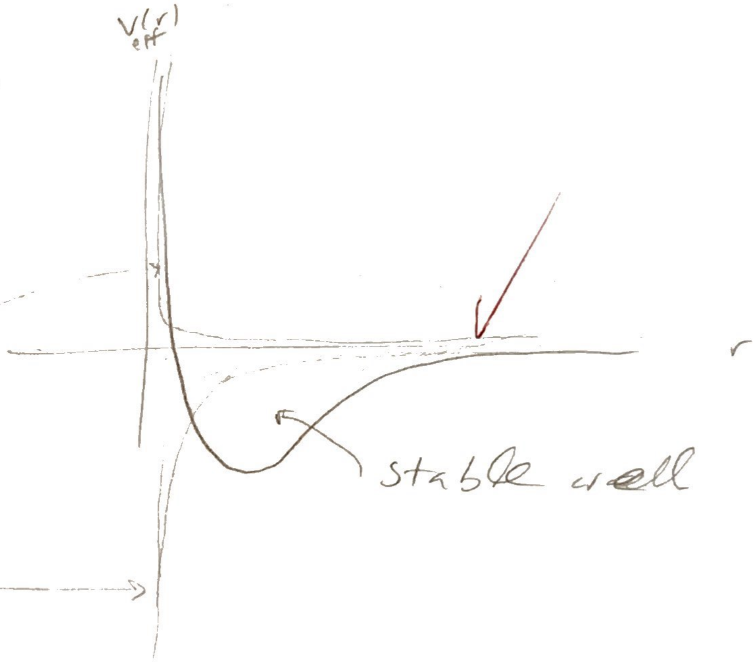
so $\boxed{U = V + \frac{mh^2}{2r^2}}$

Q.E.D.

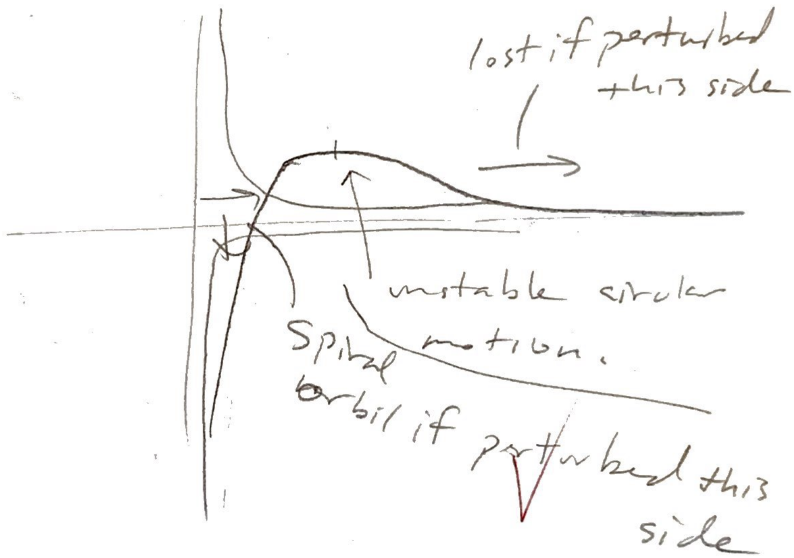
S.19 cont.

if $V(r) = \frac{-k}{r}$

$V_{eff} = \frac{-k}{r} + \frac{1}{2}\frac{mh^2}{r^2}$



if $V(r) = \frac{-k}{r^3}$



$\boxed{5.20}$ Show stability of a circular orbit is equivalent to $\frac{d^2U}{dr^2} > 0$ for $r = a$ (positive curvature)

where $U(r)$ is the effective potential $\Rightarrow$ we $U(r) = V(r) + \frac{1}{2} m \left(\frac{h}{r}\right)^2$

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graphically the stable orbits looked like

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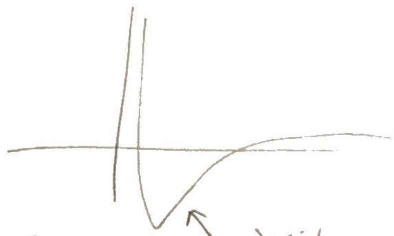

we are aware of the stability condition:

$$\mu = \frac{3}{a} f(a) + f'(a) < 0 .$$

define:

$$* \boxed{\mu + \frac{3 f(a)}{a} = -f'(a)}$$

From prob 19,

$$\boxed{u << 0}$$

$$u(r) = V(r) + \frac{1}{2} \frac{mh^2}{r^2}$$

$$U_r = V_r + \frac{1}{2} mh^2 \left[\frac{dr^{-2}}{dr} = -2r^{-3}\right]$$

$$U_{rr} = V_{rr} + \frac{1}{2} mh^2 \left[\frac{d\,-2r^{-3}}{dr} = 6r^{-4}\right] = -f'(r) + \frac{3mh^2}{r^4}$$

$$-f'(a) + \frac{3mh^2}{a^4}$$

$$U_{rr}(a) \overset{*}{=} \left[-\mu + \frac{3}{a} f(a)\right] + \frac{3mh^2}{a^4} \quad ?$$

$$= +\mu + \frac{-3}{a}\left(\frac{mh^2}{a^3}\right) + \frac{3mh^2}{a^4}$$

So $\boxed{U_{rr}(a) = u >> 0}$

please see the key

Sorry about the sloppyness

You have to show that $f(a) + \frac{a}{3} f'(a) < 0 \Rightarrow \left.\frac{d^2U}{dr^2}\right|_{r=a} > 0$

5-26

Special Relativity: particle in central field has same orbit as one in a non-relativistic frame with potential

$$U(r) = V(r) - \frac{[E - V(r)]^2}{2m_0c^2}$$

Find the apsidal angle for motion in an inverse square field

use $V(r) = -\frac{k}{r^2}$

10

$$F(r) = -\frac{\partial U(r)}{\partial r} = -\frac{\partial V}{\partial r} + \frac{\partial [E-V]^2/2m_0c^2}{\partial r}$$

$$= -\frac{\partial V}{\partial r} + \frac{1}{2m_0c^2} 2(E-V)\left[-\frac{\partial V}{\partial r}\right]$$

but $V(r) = -\frac{k}{r}$

so $f(r) = \left(+\frac{k}{r^2}\right)$

$$F(r) = f(r) + \frac{2}{2m_0c^2}[E-V]\left[f(r)\right]$$

$$= \left(+\frac{k}{r^2}\right) + \frac{1}{m_0c^2}[E-V]\left(\frac{+k}{r^2}\right)$$

$$F(r) = \frac{k}{r^2} + \frac{Ek}{r^2 m_0c^2} - \frac{k^2}{r^3 m_0c^2} \qquad F'(r) = -\frac{2k}{r^3} - \frac{2Ek}{r^3 m_0c^2} + \frac{3k^2}{r^4 m_0c^2}$$

now

$$\psi = \pi\left[3 + a\frac{f'(a)}{f(a)}\right]^{-\frac{1}{2}} = \pi\left[3 + a\left(\frac{-\frac{2k}{a^3} - \frac{2Ek}{a^3m_0c^2} + \frac{3k^2}{a^4m_0c^2}}{\frac{k}{a^2} + \frac{Ek}{a^2m_0c^2} - \frac{k^2}{a^3m_0c^2}}\right)\right]^{-\frac{1}{2}}$$

$$= \pi\left[3 + a\left(\frac{-\frac{2k}{a} - \frac{2Ek}{a\,m_0c^2} + \frac{3k^2}{a^2m_0c^2}}{k + \frac{Ek}{m_0c^2} - \frac{k^2}{a\,m_0c^2}}\right)\right]^{-\frac{1}{2}} = \pi\left[\frac{3k + \frac{3Ek}{m_0c^2} - \cancel{\frac{3k^2}{a\,m_0c^2}} - 2k - \frac{2Ek}{m_0c^2} + \cancel{\frac{3k^2}{a\,m_0c^2}}}{1k + \frac{Ek}{m_0c^2} - \frac{k^2}{a\,m_0c^2}}\right]^{-\frac{1}{2}}$$

$$= \pi\left[\frac{1 + \frac{E}{m_0c^2}}{1 + \frac{E}{m_0c^2} - \frac{k}{a\,m_0c^2}}\right]^{-\frac{1}{2}} = \pi\left[\frac{m_0c^2 + E}{m_0c^2 + E - \frac{k}{a}}\right]^{-\frac{1}{2}} = \pi\left[\frac{m_0c^2 + E - \frac{k}{a}}{m_0c^2 + E}\right]^{\frac{1}{2}}$$

$$= \pi\left[1 + \frac{-V(a)}{m_0c^2 + E}\right]^{\frac{1}{2}} \checkmark \qquad \psi = \pi\sqrt{1 + \frac{k}{a(E + m_0c^2)}}$$

5.26 Potential for an oblate spheroid

whoops! not suppose to do.

$V(r) = -\frac{k}{r}\left(1+\frac{\epsilon}{r^2}\right)$, $k = GMm$, $\epsilon = \frac{2}{5}R\Delta R$

R = equatorial radius

ΔR = equatorial − polar

Find the apsidal angle if $R = 4000$ mi $\Delta R = 13$ mi

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$$\psi = \frac{1}{2}T_r\dot{\theta} = \pi\left[3 + a\frac{f'(a)}{f(a)}\right]^{-\frac{1}{2}}$$

$$f(r) = -\frac{\partial V(r)}{\partial r} = -k\frac{d}{dr}\left(r^{-1} + \epsilon r^{-2}\right) = k r^{-2} + 2k\epsilon r^{-3}$$

$$f'(r) = -2kr^{-3} - 6k\epsilon r^{-4}$$

so

$$\psi = \pi\left[3 + a\frac{-2ka^{-3} - 6k\epsilon a^{-4}}{+ka^{-2} + 2k\epsilon a^{-3}}\right]^{-\frac{1}{2}}$$

$$= \pi\left[3 + a\frac{-2a^{-1} - 6\epsilon a^{-2}}{1 + 2\epsilon a^{-1}}\right]^{-\frac{1}{2}} = \pi\left[3 + \frac{-2 - 6\epsilon a^{-1}}{1 + 2\epsilon a^{-1}}\right]^{-\frac{1}{2}}$$

$$\psi = \pi\left[\frac{\cancel{3}^{1} + \cancel{6\epsilon a^{-1}} - \cancel{2} - \cancel{6\epsilon a^{-1}}}{1 + 2\epsilon a^{-1}}\right]^{-\frac{1}{2}} = \pi\left[1 + 2\epsilon a^{-1}\right]^{\frac{1}{2}}$$

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Good work!