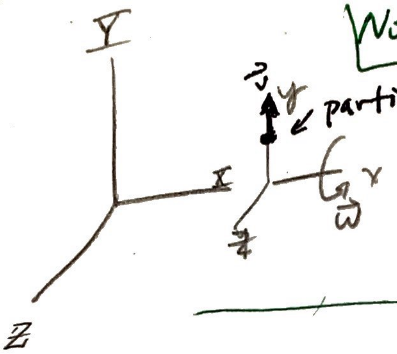
$\boxed{4.1}$

# Non-Inertial Systems




$\vec V$ = const. Find $A_{max}$ if $\vec\omega$ = constant

5/5

$$\vec A = \vec a + 2\vec\omega\times\dot{\vec r} + \dot{\vec\omega}\times\vec r + \vec\omega\times(\vec\omega\times\vec r) + \vec A_0$$

$\vec r = y\hat j$ | $\vec\omega = \omega\hat i$

$\dot{\vec r} = 0,\ \ddot{\vec r} = 0$ | $\vec A_0 = a\hat i$

$$\vec A = 0 + 0 + 2\omega\dot y(\hat i\times\hat j) + y\omega^2(\hat i\times(\hat i\times\hat j)) + \vec A_0$$

$$\vec A = a\hat i - y\omega^2\hat j + 2\hat k$$

$a = 1\ m/s^2$

$\omega = 1$

$$\boxed{\vec A = \hat i - y\hat j + 2\hat k \qquad |\vec A| = \sqrt{y^2+5}}$$

✓

4.2

a)

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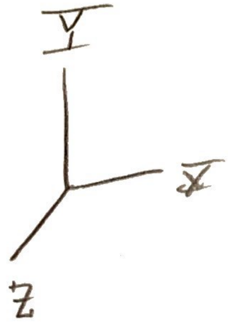
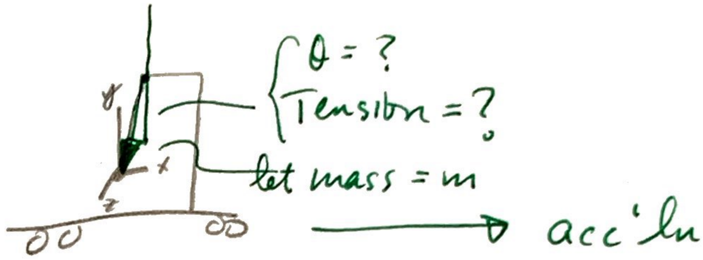


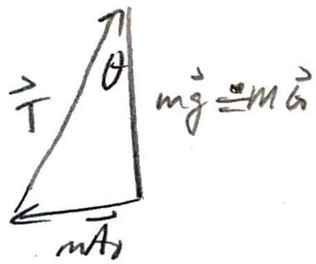

acc'ln = const $= a_0$

$$\vec{F} - m\vec{A}_0 - 2m\vec{\omega}\times\dot{\vec{r}} - m\dot{\vec{\omega}}\times\vec{r} - m\vec{\omega}\times(\vec{\omega}\times\vec{r}) = m\ddot{\vec{r}}$$

by position of axes: $\dot{\vec{r}} = 0$, $\vec{\omega} = 0$  so $\vec{F} - m\vec{A}_0 = 0$

$\vec{F}$ = true gravitational force + tension in line

or

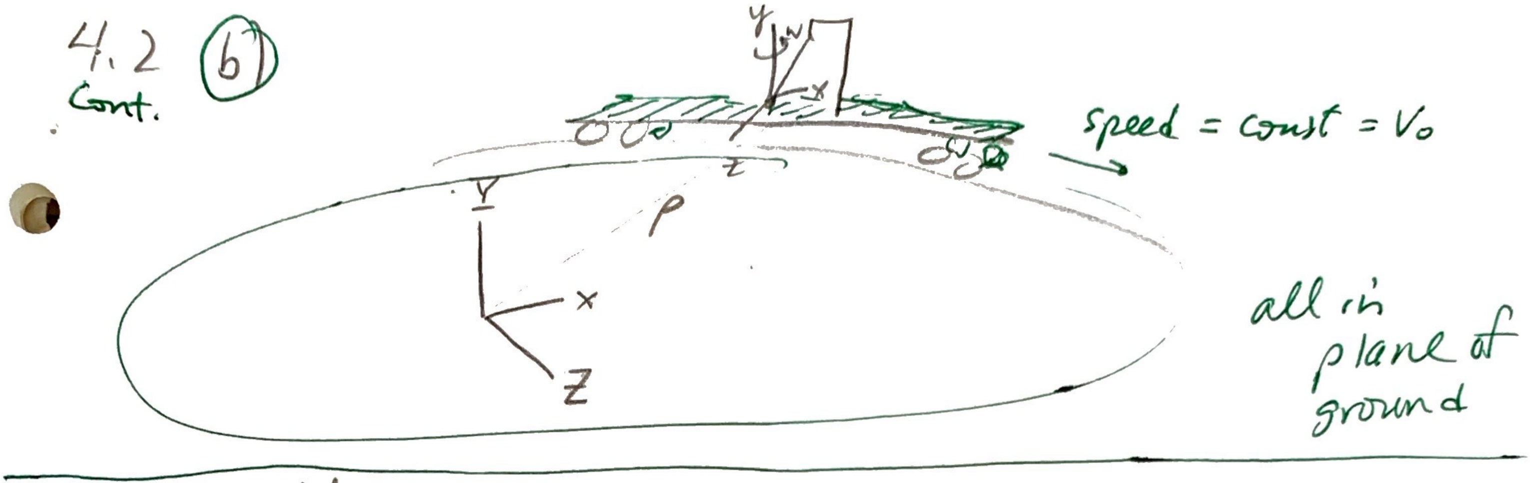



$$\tan\theta = \frac{a_0 m}{mg}$$

or $\boxed{\theta = \tan^{-1}\frac{a_0}{g}}$ ✓

$$\boxed{|\vec{T}| = \sqrt{a_0^2 + g^2}\, m}$$ ✓

4.2 (b)
Cont.

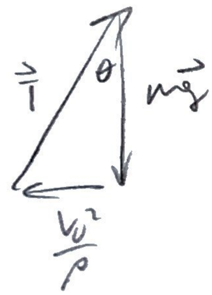



let $Oxyz$ be at bob so $\vec{r} = 0$

$m\ddot{\vec{R}}$

again $\boxed{\vec{F} - m\vec{A}_0 = 0}$ $\quad \vec{A}_0 = a_0 \hat{k} = \frac{V_0^2}{\rho}\hat{k}$

& $\boxed{\vec{F} = \vec{T} + m\vec{g}}$




so $T\cos\theta = mg$

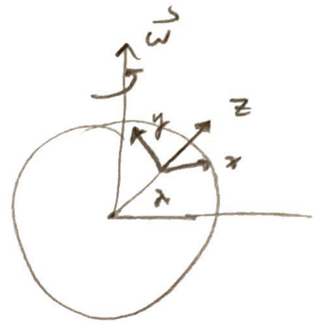
$T\sin\theta = m\frac{V_0^2}{\rho}$

$$\boxed{\theta = \tan^{-1}\frac{V_0^2}{g\rho}}$$

$$\boxed{|\vec{T}| = \sqrt{\frac{V_0^4}{\rho^2} + g^2}\; m}$$

$\boxed{4.3}$ Find Corialis force on race car $m = 10$ tons moving south at $400\,km/hr$

$$m\ddot{\vec{r}} = \vec{F} + m\vec{g} - 2m\vec{\omega}\times\dot{\vec{r}} - m\vec{\omega}\times(\vec{\omega}\times\vec{r})$$




$\dot{\vec{r}} = -400\,km/hr\ \hat{j}$

5/5

$$\vec{F}_{cor} = -2m\vec{\omega}\times\dot{\vec{r}}$$

$$\vec{\omega} = (\hat{i},\hat{j},\hat{k})\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = (\hat{i},\hat{j},\hat{k})\begin{pmatrix} 0 \\ \omega\cos\lambda \\ \omega\sin\lambda \end{pmatrix}$$

$$\dot{\vec{r}} = (\hat{i},\hat{j},\hat{k})\begin{pmatrix} 0 \\ -400\,km/hr \\ 0 \end{pmatrix} \qquad \lambda = 45^\circ$$

$$\vec{\omega}\times\dot{\vec{r}} = \hat{i}\left[0 - \omega_e\left(-400\tfrac{km}{hr}\right)\tfrac{\sqrt{2}}{2}\right] + 0\hat{j} + \hat{k}(-\omega_0\cdot 0)$$

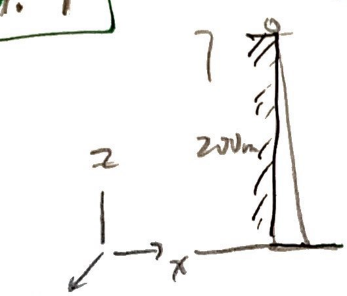
$$\vec{F}_{cor} = -2m\vec{\omega}\times\dot{\vec{r}} = -2(10\times1000\,kg)\left[7.3\times10^{-5}\tfrac{R}{s}\ \tfrac{400\times10^3 m}{3600 s}\ \tfrac{\sqrt{2}}{2}\right]\hat{i}$$

$$\boxed{\vec{F}_c = -115\,NT\,\hat{i}}$$

✓

$\boxed{4.4}$

A particle dropped from $h = 200m$. Find the deflection due to the coriolis force




$$\vec{\omega} \times \vec{r} = \hat{\imath}(\omega \dot{z} \cos\lambda - \omega \dot{y} \sin\lambda) + \hat{\jmath}\, \omega \dot{x} \sin\lambda + \hat{k}(-\omega \dot{x} \cos\lambda)$$

5/5

$$\vec{F}_{cor} = -2\vec{\omega} \times \dot{\vec{r}}\, m$$

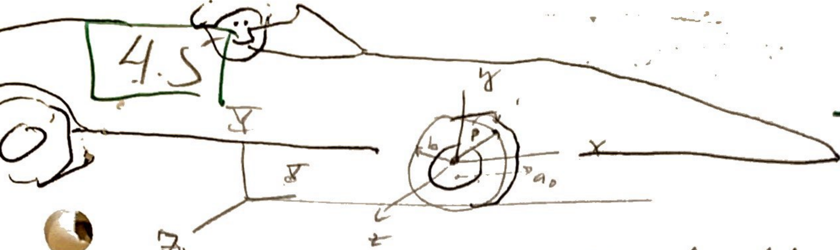
$$m \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = m \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix} - 2m\omega \begin{pmatrix} \dot{z}\cos\lambda - \dot{y}\sin\lambda \\ \dot{x}\sin\lambda \\ -\dot{x}\cos\lambda \end{pmatrix}$$

this leads to what's in the book after initial's = 0 and we have integrated

$$x_{deflection} = \frac{1}{3}\omega g t^3 \cos\lambda - \omega t^2(\cancel{\dot{z}_0}^{\,0}\cos\lambda - \cancel{\dot{y}_0}^{\,0}\sin\lambda) + \dot{x}_0 t$$

$$\vec{x} = \frac{1}{3}\left(7.3\times10^{-5}\tfrac{r}{s}\right)(9.8\,m/s^2)\left[\frac{400m}{9.8\,m/s}\right]^{1.5} = \underline{\underline{.04m}} \sim \boxed{4cm\,\hat{\imath}}$$

east.




$\longrightarrow$ acc'ln = const = $a_0$
when $V = V_0$ ...

Find the magnitude and direction of the max $\vec{A}$ of any point on the tire

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$\vec{r} = b\cos\theta\,\hat{i} + b\sin\theta\,\hat{j}$ $\qquad \vec{A}_0 = a_0\hat{i}$ $\qquad \vec{\omega} = \vec{0}$

$\dot{\vec{r}} = -b\dot{\theta}\sin\theta\,\hat{i} + b\dot{\theta}\cos\theta\,\hat{j}$

$\ddot{\vec{r}} = -b\left[\dot{\theta}^2\cos\theta + \ddot{\theta}\sin\theta\right]\hat{i} + b\left[-\dot{\theta}^2\sin\theta + \ddot{\theta}\cos\theta\right]\hat{j}$

$\underset{m\vec{A}}{\vec{F}} - m\vec{A}_0 - \cancel{2m\vec{\omega}\times\dot{\vec{r}}} - m\overset{0}{\dot{\vec{\omega}}}\times\vec{r} - m\overset{0}{\vec{\omega}}\times(\vec{\omega}\times\vec{r}) = m\ddot{\vec{r}}$

$\vec{A} - \vec{A}_0 = \ddot{\vec{r}}$ $\qquad \vec{A} = \ddot{\vec{r}} + \vec{A}_0 = \ddot{\vec{r}} + a_0\hat{i}$

or

$\vec{A} = b\dot{\theta}^2\cos\theta\,\hat{i} - b\ddot{\theta}\sin\theta\,\hat{i} - b\dot{\theta}^2\sin\theta\,\hat{j} + b\ddot{\theta}\cos\theta\,\hat{j} + a_0\hat{i}$

$|\vec{A}|^2 = a_i^2 + a_j^2 = \left[b\dot{\theta}^2\cos\theta - b\ddot{\theta}\sin\theta + a_0\right]^2 + \left[-b\dot{\theta}^2\sin\theta + b\ddot{\theta}\cos\theta\right]^2$

at the given instant

$\frac{V_0}{b} = \dot{\theta}$

$|\vec{A}_0|^2 = \left(b\frac{V_0^2}{b^2}\cos\theta - b\frac{a_0}{b}\sin\theta + a_0\right)^2 + \left(-b\frac{V_0^2}{b^2}\sin\theta + b\frac{a_0}{b}\cos\theta\right)^2$

$= a_0^2 + 2a_0^2\sin\theta + \cancel{\frac{2a_0V_0^2}{b}\cos\theta} + a_0^2\sin^2\theta + \frac{2a_0V_0^2}{b}\sin\theta\cos\theta + \frac{V_0^4}{b^2}\cos^2\theta + a_0^2\cos^2\theta$

$-\frac{2a_0V_0^2}{b}\cos\theta\sin\theta + \frac{V_0^4}{b^2}\sin^2\theta$

$2a_0^2 + \frac{V_0^4}{b^2} + 2a_0^2\sin\theta + \frac{2a_0V_0^2}{b}\cos\theta + a^2\cos^2\theta + a^2\sin^2\theta - a_0^2 \equiv 0$

$|\vec{A}| = \sqrt{\left(a\cos\theta + \frac{V_0^2}{b}\right)\left(a\sin\theta + a_0\right)^2}$ $\qquad \frac{d|A|^2}{d\theta} = 2a_0^2\cos\theta - \frac{2a_0V_0^2}{b}\sin\theta = 0$

back over

4.5 cont.

implies. $\dfrac{\sin\theta}{\cos\theta} = \dfrac{a_0 b}{V_0^2}$

$$|\vec{A}| = \sqrt{\left(a\frac{V_0^2}{a_0 b}\sin\theta + \frac{V_0^2}{b}\right)^2 \left(a\left(\frac{a_0 b}{V_0^2}\right) + a_0\right)^2}$$

$a_0^2\left[b/V_0^2 + 1\right]$ can be further simplified

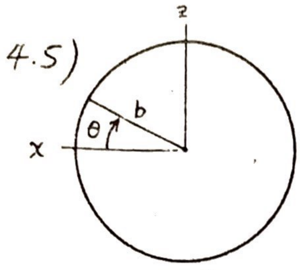
please see the key

Well, Procedure from here is

1. simplify $|\vec{A}_0|$ for mag
2. plug the $\theta_0$ back into

$$\vec{A} = \left[\frac{b\,V_0^2}{b^2}\cos\theta_0 - \frac{b\,a_0}{b}\sin\theta_0 + a_0\right]\hat{i} + \left[-\frac{V_0^2}{b}\sin\theta_0 + a_0\cos\theta_0\right]\hat{j}$$

for the direction

4.5) 


Let coordinates be moving with car but fixed in orientation so z is always vertical.

(Several other choices of coordinates could be used equally well.)

$$\vec{A}_0 = -a_0\hat{\imath}\ , \qquad \vec{\omega} = 0$$

$$\vec{A} = \vec{a} + \vec{A}_0 \qquad \vec{r} = b\cos\theta\,\hat{\imath} + b\sin\theta\,\hat{k}$$

$$\Rightarrow \vec{v} = -b\dot{\theta}\sin\theta\,\hat{\imath} - b\dot{\theta}\cos\theta\,\hat{k}$$

$$\Rightarrow \vec{a} = -b\ddot{\theta}\sin\theta\,\hat{\imath} - b\dot{\theta}^2\cos\theta\,\hat{\imath} + b\ddot{\theta}\cos\theta\,\hat{k} - b\dot{\theta}^2\sin\theta\,\hat{k}$$

$$\Rightarrow \vec{A} = -(a_0 + b\dot{\theta}^2\cos\theta + b\ddot{\theta}\sin\theta)\,\hat{\imath} + b(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta)\,\hat{k}$$

$$v_0 = b\dot{\theta}\ , \quad a_0 = b\ddot{\theta}$$

$$\Rightarrow \vec{A} = -\left(a_0 + \frac{v_0^2}{b}\cos\theta + a_0\sin\theta\right)\hat{\imath} + \left(a_0\cos\theta - \frac{v_0^2}{b}\sin\theta\right)\hat{k}$$

$$\Rightarrow A^2 = a_0^2(1+\sin\theta)^2 + \frac{v_0^4}{b^2}\cos^2\theta + 2a_0(1+\cancel{\sin\theta})\frac{v_0^2}{b}\cos\theta$$

$$+ a_0^2\cos^2\theta + \frac{v_0^4}{b^2}\sin^2\theta - \frac{2a_0v_0^2}{b}\cancel{\sin\theta}\cos\theta$$

$$= a_0^2(1 + 2\sin\theta + \sin^2\theta + \cos^2\theta) + \frac{v_0^4}{b^2} + \frac{2a_0v_0^2}{b}\cos\theta$$

$$= 2a_0^2(1+\sin\theta) + \frac{v_0^4}{b^2} + \frac{2a_0v_0^2}{b}\cos\theta$$

$$\frac{dA^2}{d\theta} = 2a_0^2\cos\theta - \frac{2a_0v_0^2}{b}\sin\theta = 0$$

$$\Rightarrow \tan\theta = \frac{a_0 b}{v_0^2} \quad \text{for max } |\vec{A}|.$$

$$\Rightarrow \vec{A}_{max} = -\left(a_0 + \frac{v_0^2}{b}\frac{v_0^2}{\sqrt{v_0^4 + a_0^2b^2}} + a_0\frac{a_0 b}{\sqrt{v_0^4+a_0^2b^2}}\right)\hat{\imath}$$

$$= -\left(a_0 + \sqrt{\frac{v_0^4}{b^2} + a_0^2}\right)\hat{\imath}$$ horizontal, forward.

$\boxed{4.7}$


acc'ln @ highest point of tire

Turning coord. system

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$$\vec{r} = \rho\hat{i} + b\cos\beta\,\hat{j} + b\sin\beta\,\hat{k}$$

$$\vec{\omega} = \frac{v}{\rho}\hat{k} \qquad \vec{A}_0 = 0$$

so

$$\dot{\vec{r}} = -\dot{\beta} b\sin\beta\,\hat{j} + b\dot{\beta}\cos\beta\,\hat{k}$$

$$\ddot{\vec{r}} = -b\left[\dot{\beta}^2\cos\beta + \ddot{\beta}\sin\beta\right]\hat{j} + b\left[-\dot{\beta}^2\sin\beta + \ddot{\beta}\cos\beta\right]\hat{k}$$

but $\ddot{\beta} = 0$ so $\ddot{\vec{r}} = b\dot{\beta}^2\cos\beta\,\hat{j} - b\dot{\beta}^2\sin\beta\,\hat{k}$

$$\vec{A} = -\dot{\beta}^2 b\cos\beta\,\hat{j} - \dot{\beta}^2 b\sin\beta\,\hat{k} + 0 + 2\frac{v}{\rho}\dot{\beta} b\sin\beta\,\hat{i} + 0$$

$\dot{\beta} = \frac{v}{b}$

$$+ \frac{v^2}{\rho^2}\rho(-\hat{i}) - \frac{bv^2}{\rho^2}\cos\beta\,\hat{j}$$

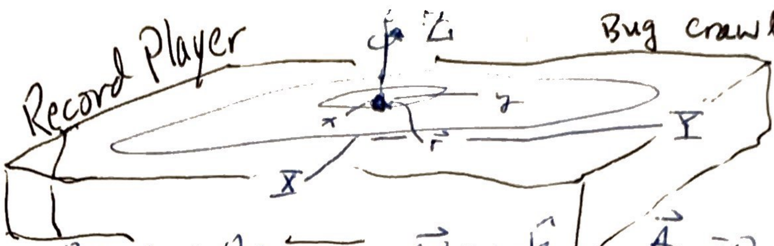
let $\beta = \frac{\pi}{2}$

$$= -\frac{v^2}{b^2}b\hat{k} + \frac{2v^2}{\rho b} + \frac{v^2}{\rho}\hat{i}$$

$$\boxed{\vec{A} = \frac{3v^2}{\rho}\hat{i} - \frac{v^2}{b}\hat{k}}$$

$$|\vec{A}| = \sqrt{\frac{9v^4}{\rho^2} + \frac{v^4}{b^2}} = \boxed{v^2\sqrt{\frac{9}{\rho^2} + \frac{1}{b^2}} = A}$$

$\boxed{4.8}$




Bug crawls in circles on moving record


$xyz$

$\vec{r} = b\cos\theta\,\hat{\imath} + b\sin\theta\,\hat{\jmath}$ $\qquad \vec{\omega} = \omega\hat{k} \qquad \vec{A}_0 = 0$

$\dot{\vec{\omega}} = \vec{0}$ $\qquad \theta = \frac{v}{b}t,\ \dot{\theta} = \frac{v}{b},\ \ddot{\theta} = 0$

$\dot{\vec{r}} = -b\dot{\theta}\sin\theta\,\hat{\imath} + b\dot{\theta}\cos\theta\,\hat{\jmath}$

$\ddot{\vec{r}} = (-b\dot{\theta}^2\cos\theta - b\ddot{\theta}\sin\theta)\hat{\imath} + (b\ddot{\theta}\cos\theta - b\dot{\theta}^2\sin\theta)\hat{\jmath}$

$\ddot{\vec{R}} = \vec{A} = \ddot{\vec{r}} + \vec{A}_0 + 2\vec{\omega}\times\dot{\vec{r}} + \dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r})$

$\ddot{\vec{R}} = -b\begin{pmatrix}\dot{\theta}^2\cos\theta\,\hat{\imath} \\ \dot{\theta}^2\sin\theta\,\hat{\jmath}\end{pmatrix} + 0 + 2\dot{\theta}\omega b(\underbrace{\hat{k}\times(-\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath})}_{[-\sin\theta\,\hat{\jmath} - \cos\theta\,\hat{\imath}]}) + \omega^2 b(\hat{k}\times(\underbrace{\hat{k}\times(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})}_{[\cos\theta\,\hat{\jmath} - \sin\theta\,\hat{\imath}]})) $

$[-\cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath}]$

$$\boxed{\begin{aligned}\vec{A} = &(-b\dot{\theta}^2\cos\theta - 2\omega b\dot{\theta}\cos\theta - \omega^2 b\cos\theta)\hat{\imath} + \\ &(-b\dot{\theta}^2\sin\theta - 2\omega b\dot{\theta}\sin\theta - \omega^2 b\sin\theta)\hat{\jmath}\end{aligned}}$$


$\vec{A}$ points in to center

$\boxed{\vec{F}_f = m\vec{A} = \mu m g \dfrac{\vec{A}}{|\vec{A}|}}$

(a) case #1

let $\underline{\vec{A}\,(v=\omega b)}$ $= \begin{vmatrix}(-bv^2/b^2 - 2\omega b\frac{v}{b} - \omega^2 b)\cos\theta\,\hat{\imath} + \\ (-bv^2/b^2 - 2\omega b\frac{v}{b} - \omega^2 b)\sin\theta\,\hat{\jmath}\end{vmatrix} = -\begin{vmatrix}(\frac{v^2}{b} + 2\omega v + \omega^2 b)\cos\theta\,\hat{\imath} \\ (\frac{v^2}{b} + 2\omega v + \omega^2 b)\cos\theta\,\hat{\jmath}\end{vmatrix} = \vec{A}$

op

$\vec{A} = -(\hat{\imath}, \hat{\jmath})\begin{pmatrix}(b\omega^2 + 2\omega^2 b + \omega^2 b)\cos\theta \\ 4b\omega^2\sin\theta\end{pmatrix} = \underline{\underline{-4b\omega^2[\underbrace{\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}}_{\hat{e}_r}]}}$ ✓

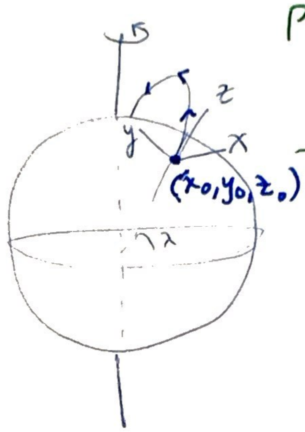
(b) case #2

let $\underline{\vec{A}\,(v=-\omega b)}$ $= -(b\omega^2 - 2\omega^2 b + \omega^2 b)\cos\theta\,\hat{\imath} + 0\sin\theta\,\hat{\jmath} = \underline{\underline{\vec{0}}}$ ✓

$$\boxed{\begin{aligned}&\vec{F}_f(v=\omega b) = \vec{A}m \\ &\vec{F}_f(v=-\omega b) = 0\end{aligned}}$$

$\boxed{4.10}$ projectile shot vertically. where does it land?

$v_0 \to \dot{z}_0, \dot{x}_0 = 0, \dot{y}_0 = 0$

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we have

$$\begin{cases} x = \frac{1}{3}\omega g t^3 \cos\lambda - \omega t^2(\dot{z}_0 \cos\lambda - \dot{y}_0 \sin\lambda) + \dot{x}_0 t \\ y = \dot{y}_0 t - \omega \dot{x}_0 t^2 \sin\lambda \\ z = -\frac{1}{2} g t^2 + \dot{z}_0 t + \omega \dot{x}_0 t^2 \cos\lambda. \end{cases}$$

$$z(t) = -\frac{1}{2} g t^2 + v_0 t + \omega(0) t^2 \cos\lambda$$

(time up and down)

$$z(t_{all}) \equiv 0 = -\frac{1}{2} g t_a^2 + v_0 t_a = \underset{0}{t_{all}} \underbrace{\left(v_0 - \frac{g t_a}{2}\right)}_{0} = 0 \qquad \boxed{\frac{2v_0}{g} = t_{all}}$$

$$x(t_{all}) = \frac{1}{3} g\omega \left(\frac{2v_0}{g}\right)^3 \cos\lambda - \omega\left(\frac{2v_0}{g}\right)^2 (v_0 \cos\lambda)$$

$$= \frac{8}{3}\frac{\omega v_0^3 \cos\lambda}{g^2} - \frac{12\omega v_0^3 \cos\lambda}{3\, g^2} = \boxed{-\frac{4}{3}\frac{\omega v_0^3 \cos\lambda}{g^2} = x_{all}}$$

$$\boxed{y(t_{all}) = 0 - 0 = 0}$$

lands $\to \boxed{(x, y, z) = \left(-\frac{4}{3}\frac{\omega v_0^3 \cos\lambda}{g^2}, 0, 0\right)}$ ✓

$\boxed{4.12}$ ODE of charged particle in an $\vec{E}$ & $\vec{B}$ field is

$$m\ddot{\vec{r}} = q\vec{E} + q\vec{v}\times\vec{B} \qquad \text{(in inertial frame)}$$

show $m\ddot{\vec{r}} = q\vec{E}$ if a coordinate syst. rotates $\vec{\omega} = \left(\frac{q}{2m}\right)\vec{B}$ where $B$ is small enough $B^2$ terms are forgotten

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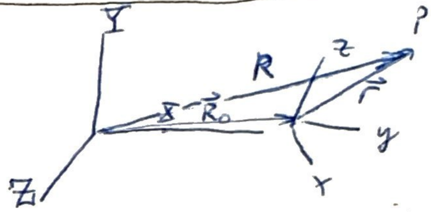


$$\left.\frac{d\vec{q}}{dt}\right|_{fixed} = \dot{\vec{q}} + \vec{\omega}\times\vec{q}$$

Coriolis Force (as seen in fixed system)

$$m\ddot{\vec{r}} = \vec{F} - m\vec{A}_0 - 2m\vec{\omega}\times\dot{\vec{r}} - m\dot{\vec{\omega}}\times\vec{r} - \vec{\omega}\times(\vec{\omega}\times\vec{r})m$$

Force seen from Moving origin.

physical force seen from fixed.

origin acceleration

Transverse Force (as seen in fixed)

Centrifugal Force (as seen in fixed)

$$m\ddot{\vec{R}} = \overbrace{\left[q\vec{E} \times q\vec{v}\times\vec{B}\right]} = m\ddot{\vec{r}} + m\vec{A}_0 + 2m\vec{\omega}\times\dot{\vec{r}} + \dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$\vec{\omega} = \frac{q}{2m}\vec{B}$, no translation $\vec{A}_0 = \vec{0}$

assume negligible

if $\vec{B} = \vec{const}$. then $\dot{\vec{\omega}} = 0$

$$= m\ddot{\vec{r}} + 0 + 2m\left(\frac{q}{2m}\right)\vec{B}\times\dot{\vec{r}} + 0 + \cancel{\frac{q^2}{(2m)^2}\vec{B}\times(\vec{B}\times\vec{r})}$$

so

$$q\vec{E} \times q\vec{v}\times\vec{B} \approx m\ddot{\vec{r}} + q\vec{B}\times\vec{v}$$

or $\boxed{q\vec{E} = m\ddot{\vec{r}}}$

OXYZ

## Lagrangian Mechanics

**9.1** Find the O.D.E's of motion of a projectile in a uniform gravitational field (No air resistance)


5/5

In cartesian coordinates

$$T = \frac{1}{2} m v^2 = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

$$V = -mgz$$

Lagranges function $L = T - V = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - mgz$

use

$$\dot{\left(\frac{\partial L}{\partial \dot{x}}\right)} = \frac{\partial L}{\partial x} \quad ; \quad \dot{\left(\frac{\partial L}{\partial \dot{y}}\right)} = \frac{\partial L}{\partial y} \quad ; \quad \dot{\left(\frac{\partial L}{\partial \dot{z}}\right)} = \frac{\partial L}{\partial z}$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} \,,\; \frac{\partial L}{\partial x} = 0 \quad ; \quad \frac{\partial L}{\partial \dot{y}} = m\dot{y} \,,\; \frac{\partial L}{\partial y} = 0 \quad ; \quad \frac{\partial L}{\partial \dot{z}} = m\dot{z} \,,\; \frac{\partial L}{\partial z} = -mg$$

$$\Rightarrow \boxed{m\ddot{x} = 0 \,,\; m\ddot{y} = 0 \quad m\ddot{z} = -mg}$$