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PHYS 321

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ASSGN. #1
Ch 1: 18, 19, 20, 22, 25, 26

Chpt 1: $\boxed{\text{Vectors}}$

$\boxed{1.18}$

Let speed be described by $v = At^2$

For what t does the acceleration vector $\vec{a}$ make an angle of 45° with the velocity vector $\vec{v}$.

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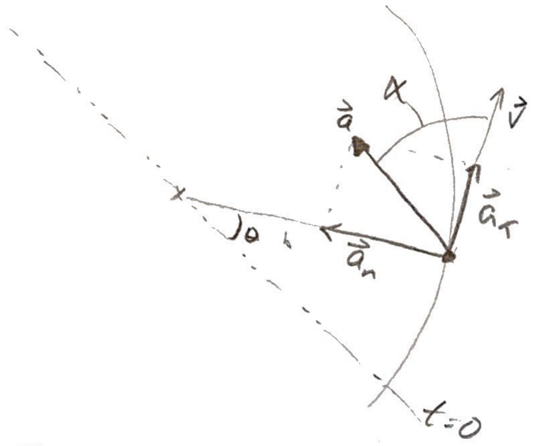

The velocity vector is always tangent to circle (path.)

$\vec{v} = v\vec{\tau}$ so I'll compare components in the acceleration to find α.

$$\vec{a} = \dot{\vec{v}} = \frac{d\, v\vec{\tau}}{dt} = \dot{v}\vec{\tau} + v\dot{\vec{\tau}}$$

but $\dot{\vec{\tau}} = \vec{n}\frac{v}{\rho}$

ρ is the radius of curvature, in this case constant $\rho = b$

so $\vec{a} = \dot{v}\vec{\tau} + \frac{v^2}{b}\vec{n}$ $\qquad \vec{a}(t) = (2At)\vec{\tau} + \frac{(At^2)^2}{b}\vec{n}$

for α to $= 45°$ $|\vec{a}_\tau| = |\vec{a}_n|$ or $2At = \frac{A^2t^4}{b}$

$A^2t^4 - 2At = 0$

$At(At^3 - 2) = 0$

$t = \left(\frac{2}{A}\right)^{1/3}$

$t = \sqrt[3]{\frac{2b}{A}}$

$\boxed{t = 0^+}$

$\boxed{t = \frac{\sqrt[3]{2b}}{\sqrt[3]{A}}}$

1.19

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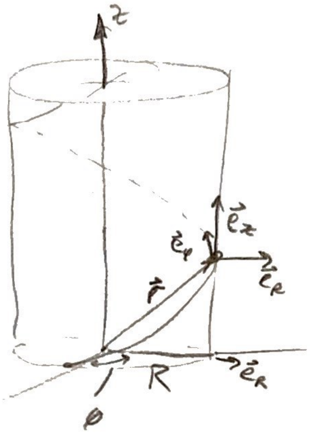


$R = b$ $\qquad \dot{R} = 0$ $\qquad \ddot{R} = 0$

$\varphi = wt$ $\qquad \dot{\varphi} = w$ $\qquad$ (assume w = const) $\ddot{\varphi} = 0$

$z = ct^2$ $\qquad \dot{z} = c2t$ $\qquad \ddot{z} = 2c$

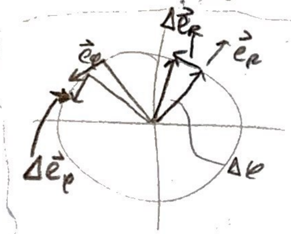
Find the speed, magnitude of acceleration as function of t.

Speed

$$\vec{r} = R\vec{e}_R + z\vec{e}_z$$

$$\vec{v} = \dot{\vec{r}} = \frac{d R\vec{e}_R}{dt} + \frac{d z\vec{e}_z}{dt} = \dot{R}\vec{e}_R + R\dot{\vec{e}}_R + \dot{z}\vec{e}_z + z\dot{\vec{e}}_z$$

$$\frac{d\vec{e}_z}{dt} = 0, \quad \dot{\vec{e}}_R$$



$\Delta\vec{e}_R \sim \Delta\varphi\,\vec{e}_\varphi$ $\quad$ or $\quad d\vec{e}_R = d\varphi\,\vec{e}_\varphi$

similarly

$\Delta\vec{e}_\varphi \sim |\vec{e}_\varphi \Delta\varphi|(-\vec{e}_R)$ $\quad$ or $\quad d\vec{e}_\varphi = -d\varphi\,\vec{e}_R$

so $\quad \dot{\vec{e}}_R = \dot{\varphi}\vec{e}_\varphi$

$$\vec{v} = \dot{R}\vec{e}_R + R(\dot{\varphi}\vec{e}_\varphi) + \dot{z}\vec{e}_z = \dot{R}\vec{e}_R + R\dot{\varphi}\vec{e}_\varphi + \dot{z}\vec{e}_z$$

$$\text{speed} = |\vec{v}| = \sqrt{\dot{R}^2 + (R\dot{\varphi})^2 + \dot{z}^2} = \sqrt{0^2 + (bw)^2 + (2ct)^2} = (b^2w^2 + 4c^2t^2)^{1/2}$$

$$\boxed{v(t) = (b^2w^2 + 4c^2t^2)^{1/2}}$$

✓

acceleration

$$\vec{v} = \dot{R}\vec{e}_R + R\dot{\varphi}\vec{e}_\varphi + \dot{z}\vec{e}_z$$

$$\vec{a} = \dot{\vec{v}} = \ddot{R}\vec{e}_R + \dot{R}\dot{\vec{e}}_R + \dot{R}\dot{\varphi}\vec{e}_\varphi + R\ddot{\varphi}\vec{e}_\varphi + R\dot{\varphi}\dot{\vec{e}}_\varphi + \ddot{z}\vec{e}_z + \dot{z}\dot{\vec{e}}_z$$

$$\vec{a} = \ddot{R}\vec{e}_R + \dot{R}\dot{\varphi}\vec{e}_\varphi + \dot{R}\dot{\varphi}\vec{e}_\varphi + R\ddot{\varphi}\vec{e}_\varphi + R\dot{\varphi}(-\dot{\varphi}\vec{e}_R) + \ddot{z}\vec{e}_z + 0$$

or $\quad \vec{a} = (\ddot{R} - R\dot{\varphi}^2)\vec{e}_R + (2\dot{R}\dot{\varphi} + R\ddot{\varphi})\vec{e}_\varphi + \ddot{z}\vec{e}_z$

$$|\vec{a}| = \sqrt{(0 - bw^2)^2 + (0 + b\cdot 0)^2 + (2c)^2} = (b^2w^4 + 4c^2)^{1/2}$$

so $\quad \boxed{a(t) = (b^2w^4 + 4c^2)^{1/2}}$

✓

$\boxed{1.20}$ 5 | 5

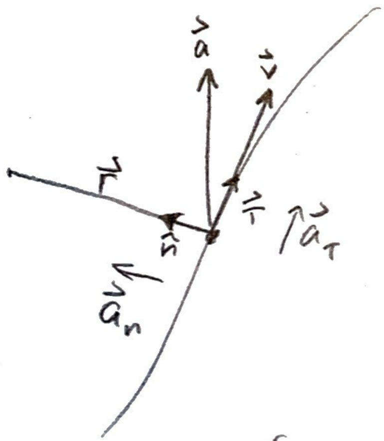
Show that $|\vec{a}_T| = \dfrac{\vec{a}\cdot\vec{v}}{|\vec{v}|}$ and $|\vec{a}_n| = (a^2 - a_T^2)^{1/2}$ where

$\vec{a} = \vec{a}_T + \vec{a}_n$.

① $\vec{a} = \dot{v}\hat{T} + \dfrac{v^2}{\rho}\hat{n}$

② also note $|\vec{a}| = \sqrt{|a_T|^2 + |\vec{a}_n|^2}$ or $a^2 = a_T^2 + a_n^2$

so $a_n^2 = a^2 - a_T^2$ $\boxed{a = (a^2 - a_T^2)^{1/2}}$



$\vec{V} = V\hat{T}$

by virtue of the scalar product $\vec{a}\cdot\hat{T} = |\vec{a}_T|$

from $\vec{v} = v\hat{T}$ we get $\hat{T} = \dfrac{\vec{V}}{V} = \dfrac{\vec{V}}{|\vec{V}|}$

so $\vec{a}\cdot\hat{T} = |\vec{a}_T|$

$$\boxed{\frac{\vec{a}\cdot\vec{V}}{|\vec{V}|} = |\vec{a}_T|}$$

1.$\boxed{22}$

Prove $\vec{v}\cdot\vec{a} = v\dot{v}$ (showing $\vec{v}\perp\vec{a}$ if v=const.)

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$\vec{v}\cdot\vec{v} = v^2$ $\qquad d(\vec{v}\cdot\vec{v}) = dv^2$ $\quad$ or $\quad (d\vec{v})\cdot\vec{v} + \vec{v}\cdot d\vec{v} = 2v\,dv$

$$\cancel{2}\,d\vec{v}\cdot\vec{v} = \cancel{2}\,v\,dv$$

$\frac{d\vec{v}}{dt} = \vec{a}$ $\quad$ so $\quad \boxed{\vec{a}\cdot\vec{v} = v\frac{dv}{dt}}$ ✓ $\quad$ if $\dot{v}=0 \Rightarrow \vec{v}\perp\vec{a}$ or $\vec{a}=0$

1.$\boxed{25}$ Knowing $\hat{T} = \frac{\vec{v}}{|\vec{v}|}$ find an expression for $\hat{n}$ in terms of $\vec{a}, a, \vec{v}, v, \&\ \dot{v}$

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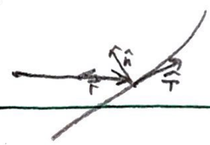


(we know from prob. 22 $\quad \vec{v}\cdot\vec{a} = v\dot{v}$)

lets start with $\quad \vec{a} = \vec{a}_T + \vec{a}_n = (\vec{a}\cdot\hat{T})\hat{T} + a_n\hat{n}$

solve for $\hat{n}$ $\qquad \hat{n} = \frac{\vec{a} - (\vec{a}\cdot\hat{T})\hat{T}}{a_n}$ $\quad$ or $\quad \frac{\vec{a} - (\vec{a}\cdot\hat{T})\hat{T}}{(a^2 - a_T^2)^{1/2}}$

but $\hat{T} = \frac{\vec{v}}{v}$ and $a_T = \vec{a}\cdot\hat{T}$ so

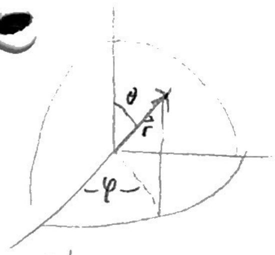
$$\hat{n} = \frac{\vec{a} - \left(\frac{\vec{a}\cdot\vec{v}}{v}\right)\hat{T}}{\left(a^2 - (\vec{a}\cdot\hat{T})^2\right)^{1/2}} = \frac{\vec{a} - \left(\frac{\vec{a}\cdot\vec{v}}{v}\right)\frac{\vec{v}}{v}}{\left(a^2 - \left(\frac{\vec{a}\cdot\vec{v}}{v}\right)^2\right)^{1/2}}$$ but $\vec{v}\cdot\vec{a} = v\dot{v}$

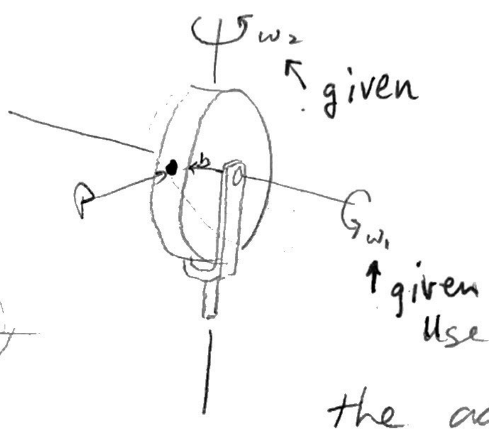
therefore

$$\hat{n} = \frac{\vec{a} - \frac{\dot{v}\vec{v}}{v}}{(a^2 - \dot{v}^2)^{1/2}} = \boxed{\frac{\vec{a} - \left(\frac{\dot{v}}{v}\right)\vec{v}}{(a^2-\dot{v}^2)^{1/2}} = \hat{n}}$$ ✓

$$\frac{\vec{a} - \frac{\dot{v}}{v}\vec{v}}{(a^2 - \dot{v}^2)^{1/2}}$$

1. $\boxed{26}$





$r = b \qquad \dot r = 0 \qquad \ddot r = 0$

$\theta = \omega_1 t \qquad \dot\theta = \omega_1 \qquad \ddot\theta = 0$

$\varphi = \omega_2 t \qquad \dot\varphi = \omega_2 \qquad \ddot\varphi = 0$

Use spherical coordinates to find the acceleration of any point on the rim of the wheel.

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text equation 1.60 is already derived so we'll use it.

$$\vec a = (\ddot r - r\dot\varphi^2 \sin^2\theta - r\dot\theta^2)\hat e_r + (r\ddot\theta + 2\dot r\dot\theta - r\dot\varphi^2 \sin\theta\cos\theta)\hat e_\theta$$
$$+ (r\ddot\varphi\sin\theta + 2\dot r\dot\varphi\sin\theta + 2r\dot\theta\dot\varphi\cos\theta)\hat e_\varphi$$

plugging in -

$$\vec a = (0 - b\omega_2^2 \sin^2\omega_1 t - b\omega_1^2)\hat e_r + (r(0) + 2\cdot(0) - b\omega_2^2 \sin\omega_1 t\cos\omega_1 t)\hat e_\theta$$
$$+ (b(0) + 2(0) + 2b\omega_1\omega_2\cos\omega_1 t)\hat e_\varphi$$

$$\boxed{\vec a = -b(\omega_2^2 \sin^2\omega_1 t + \omega_1^2)\hat e_r + (-b\omega_2^2 \sin\omega_1 t\cos\omega_1 t)\hat e_\theta + (2b\omega_1\omega_2\cos\omega_1 t)\hat e_\varphi}$$

assuming highest means a 'p' on the wheel located on the verticle axis.

here $\theta = 0 + 2n\pi$, $r = b$, $\varphi = \omega_2 t$

so $\vec a = -b(\omega_2^2 \sin^2 0 + \omega_1^2)\hat e_r + (-b\omega_2^2 \sin 0\cos 0)\hat e_\theta + (2b\omega_1\omega_2\cos 0)\hat e_\varphi$

$$= \boxed{(-b\omega_1^2)\hat e_r + 2b\omega_1\omega_2\hat e_\varphi = \vec a\,(\theta = 0)} ✓$$

This checks on $a_\theta \hat e_\theta = 0$