PHYSICS 321 EXAMINATION 1 October 2, 1981

Chpt 1-4:

Closed book. Please write on one side of paper only. Show all work.

10 1. Find the components of velocity in spherical coordinates. Be explicit in your arguments.

20 2. Write down, explaining each term and pointing out use of Newton's laws, the differential equations of motion in component form (i.e. do not leave as a single vector equation) for a single particle moving in the following force fields:

a. Uniform gravitational field with viscous air resistance proportional to speed squared (3-dimensional motion).

b. Simple harmonic oscillation with a constant damping force and cosine driving force (1-dimensional motion).

c. Uniform magnetic field $\vec{B}=B\hat{k}$ (3-dimensional motion. Particle has charge q.)

d. Uniform gravitational field with motion constrained to a circle (simple pendulum).

DO NOT SOLVE THESE DIFFERENTIAL EQUATIONS

20 3. If F=-kv for rectilinear motion of a particle, find x(t). The particle is initially moving with speed v_0 at x=0.

20 4. The motion of the 1-dimensional harmonic oscillator with damping force $-m\gamma\dot{x}$ and driving force $F_0 \cos\omega t$ is given by

$$x(t) = \frac{F_0}{\sqrt{(k-m\omega^2)^2 + m^2\gamma^2\omega^2}} \cos\left(\omega t - \tan^{-1}\frac{m\gamma\omega}{k-m\omega^2}\right),$$

in steady state.

a. What is the physical significance of the parameters

$$\gamma,\ \sqrt{\frac{k}{m}},\ \omega_r = \sqrt{\frac{k}{m} - \frac{\gamma^2}{2}}\ ?$$

b. What is "resonance" physically? When does it occur?

c. Is this a "conservative" system? Why or why not?

d. The steady state solution is a particular solution of the differential equation. What is the physical nature of the solution to the homogeneous equation and why can we often ignore it?

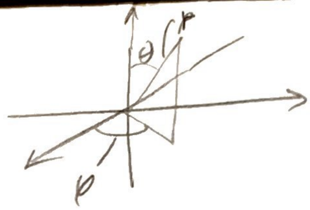
transient solution that decays over time.

POSSIBLY USEFUL INFORMATION:

In polar coordinates, $a_r = \ddot{r} - r\dot{\theta}^2$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

EXAM 1



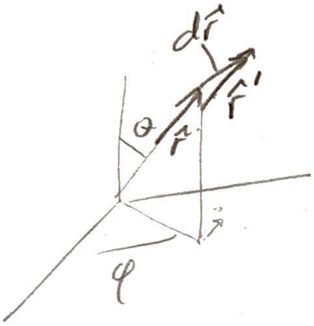
$\frac{45}{60} = 75\%$

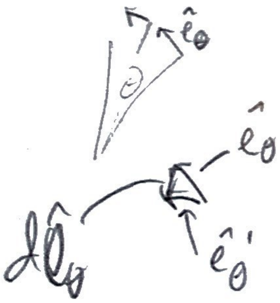
1. $\vec{v} = \dot{\vec{r}}$ ✓

$$\cancel{r = r\hat{e}_r + \theta\hat{e}_\theta + \varphi\hat{e}_\varphi}$$

$$\cancel{\frac{d\vec{r}}{dt} = \dot{r}\hat{e}_r + r\frac{d\hat{e}_r}{dt} + \dot{\theta}\hat{e}_\theta + \theta\frac{d\hat{e}_\theta}{dt} + \dot{\varphi}\hat{e}_\varphi + \varphi\frac{d\hat{e}_\varphi}{dt}}$$

$\frac{d\hat{e}_r}{dt}$





$d\hat{e}_\theta = -r$

NEW!

$\vec{r} = r\hat{e}_r$ ✓

$\frac{d\vec{r}}{dt} = \vec{v} = \dot{r}\hat{e}_r + r\frac{d\hat{e}_r}{dt}$ ✓

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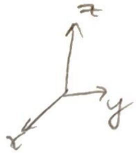
$d\hat{e}_r =$

last attempt

$\boxed{2a.}$ $F_R \propto v^2$



by Newt. 2nd : the eq. of motion $\vec{F}_{TOTAL} = \vec{F}_{resistance} + \vec{F}_{gravity}$

$$F_{resist} = -(\text{sig } v)\, v^2 \gamma m\, \hat{v}$$ redundant.

$$\vec{F}_{gravity} = mg\hat{k}$$ Which direction is up?

by 2nd law: $F_T = m\ddot{\vec{r}}$

$$\vec{F}_T = \vec{F}_R + \vec{F}_g \qquad m\ddot{\vec{r}} = -\cancel{(\text{sig } v)}\,\gamma m v^2 \hat{v} + mg\hat{k}$$

$$m(\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}) = -\cancel{(\text{sig } v)}\,\gamma m v^2 \frac{\dot{\vec{r}}}{\|\dot{\vec{r}}\|} + mg\hat{k} \qquad \|\dot{\vec{r}}\| = v$$

$$= -\cancel{(\text{sig } v)}\,\gamma m v^{\cancel{2}} \frac{\dot{\vec{r}}}{\cancel{v}} + mg\hat{k}$$

$$= -\cancel{(\text{sig } v)}\,\gamma m v(\dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}) + mg\hat{k}$$

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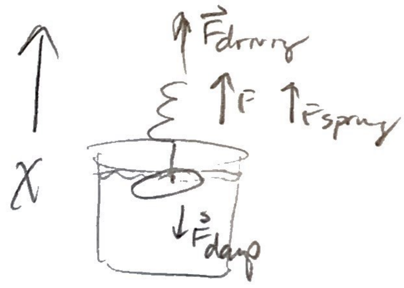
component form

$$\begin{cases} m\ddot{x} = -\cancel{(\text{sig } v)}\,\gamma m v \dot{x} \\ m\ddot{y} = -\cancel{(\text{sig } v)}\,\gamma m v \dot{y} \\ m\ddot{z} = -\cancel{(\text{sig } v)}\,\gamma m v \dot{z} \overset{?}{+} mg \end{cases}$$

$\boxed{2.b}$ SMO, const. damp, cosine driving F.

by Newtons 2nd: $\vec{F}_T = \vec{F}_{damp} + \vec{F}_{spring} + \vec{F}_{driving}$



$$\left\{\begin{array}{l} \vec{F}_{damp} = -c\dot{x} \quad \text{constant} \\ F_{spring} = -kx \\ F_{driving} = F_0 \cos \omega t \\ F_{TOTAL} = m\ddot{x} \end{array}\right.$$

so apply the 2nd set to the 1st eq.

$$m\ddot{x} = -c\dot{x} - kx + F_0 \cos \omega t$$

or $$\underline{m\ddot{x} + c\dot{x} + kx = F_0 \cos \omega t.}$$

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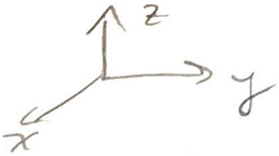
$\boxed{2.c}$ $\vec{B} = B\hat{k}$



2^{nd} law; $\vec{F}_T = \vec{F}_{mag}$

$\underline{\vec{F}_{mag} = q(\vec{v} \times \vec{B})}$ given

$$m\ddot{\vec{r}} = q(\dot{\vec{r}} \times \vec{B})$$

$$\dot{\vec{r}} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dot{x} & \dot{y} & \dot{z} \\ 0 & 0 & B \end{vmatrix} = B(\dot{y}\hat{i} - \dot{x}\hat{j})$$

so $m\ddot{\vec{r}} = q(B)(\dot{y}\hat{i} - \dot{x}\hat{j})$

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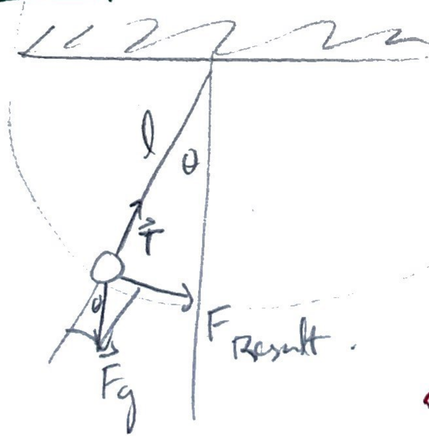
$$\underline{m(\ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}) = qB(\dot{y}\hat{i} - \dot{x}\hat{j})}$$

separating components

$$\begin{cases} m\ddot{x} = qB\dot{y} \\ m\ddot{y} = -qB\dot{x} \\ m\ddot{z} = 0 \end{cases}$$

$\boxed{2.d}$ Simple pendulum.



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again by the 2^{nd} law

$$\vec{F}_R = \vec{T} + \vec{F}_g$$

in polar coordinates ?

$$\underline{F_R \hat{e}_\theta = -mg\sin\theta\,\hat{e}_\theta}$$

~~$\vec{F}_R \perp \vec{T}$ so no component of $\vec{T}$ in $\vec{F}_R$~~ Not true.

?

$$\underline{\underline{ml\ddot{\theta} + mg\sin\theta = 0}}$$

$\boxed{3.}$ $F = -kv$, find $x(t)$ v_0 @ $x=0$

$$m\frac{dv}{dt} = -kv$$

$$\int_{v_0}^{v}\frac{dv}{v} = \int_{t_0}^{t}-\frac{k}{m}\,dt$$

$$\ln|v|\Big|_{v_0}^{v} = -\frac{k}{m}t\Big|_{t_0}^{t} \Rightarrow \ln\left|\frac{v}{v_0}\right| = -\frac{k}{m}(t-t_0)$$

$$v = v_0 e^{-\frac{k}{m}(t-t_0)}$$

$t_0 = 0.$

so $$\int_{x_0}^{x} dx = \int_{t_0}^{t} v_0 e^{-\frac{k}{m}(t-t_0)}\,dt \qquad \dot{x} \equiv$$

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$$(x - x_0) = v_0\left(-\frac{k}{m}\right)e^{-\frac{k}{m}(t-t_0)}\Big|_{t_0}^{t}$$

$$= -\frac{v_0 k}{m}\left[e^{-k/m(t-t_0)} - \cancel{e^{-k/m(t_0-t_0)}}\right] \quad 1$$

$$x - x_0 = -\frac{v_0 k}{m}\left[e^{-k/m(t-t_0)} - 1\right]$$

$$\boxed{x = -\frac{v_0 k}{m}\left[e^{-\frac{k}{m}(t-t_0)} - 1\right] + x_0}$$

I assume initially $\Rightarrow$ $t_0 = 0$ so $x_0 = 0$

$$\therefore \quad x(t) = +\frac{v_0}{m}k\left[1 - e^{-k/m\,t}\right] \Longleftarrow$$

QED

$\boxed{4.}$ $\boxed{x(t) = \frac{F_0}{\sqrt{(k-m\omega^2)^2 + m^2\gamma^2\omega^2}} \cos\left(\omega t - \tan^{-1} \frac{m\gamma\omega}{k-m\omega^2}\right)}$

(a.) what is the physical significance of

γ, $\sqrt{\frac{k}{m}}$, $\omega_r = \sqrt{\frac{k}{m} - \frac{\gamma^2}{2}}$

1. $\sqrt{\frac{k}{m}}$: angular frequency of undamped, undriven system.

2. γ : $\left(\frac{m}{s}\right) kg \left(\frac{1}{s}\right)$

γ is in the characteristic time.

in unit times, a non driven, damped system decays $\frac{1}{e}$ of the original value (depending on that time γ)

3. $\omega_r = \sqrt{\frac{k}{m} - \frac{\gamma^2}{2}}$ this is the resonant frequency for which the amplitude is at the greatest possible value for given γ, k, m's, if its driven at ωr.

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$\boxed{4.b.}$ resonance: the value of the driving frequency which causes the energy to transfer from driving mechanism to spring system at the greatest amounts.

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$\boxed{\text{nat freq. } \omega_1 = \sqrt{\frac{k}{m} - \gamma^2}}$

$\boxed{\text{undamped nat. freq. } \omega = \sqrt{k/m}}$

$\boxed{4.c.}$ This is not a conservative system due to the damping effects. (energy is lost over each ~~certain~~ period). if $\gamma = 0$ then all E goes into spring and it 'blows up'!

$-kx - b\dot{x} + F_0 \cos(\omega t + \theta_0)$

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$F \neq F(x)$

but

$F(\dot{x}, x, t)$

4.d.

The eq. becomes homogeneous when their is no driving force. It yields underdamped critically damped and overdamped solns.

But when the system is driven at a certian frequency all parts of that system are moving (maybe out of phase) at that frequency. So the particular part of the soln. doesn't add new info. see key.

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