*21–15 Resonance in AC Circuits

The rms current in an LRC series circuit is given by (see Eqs. 21–14, 21–15, 21–11b, and 21–12b):

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}. \qquad \textbf{(21–18)}$$

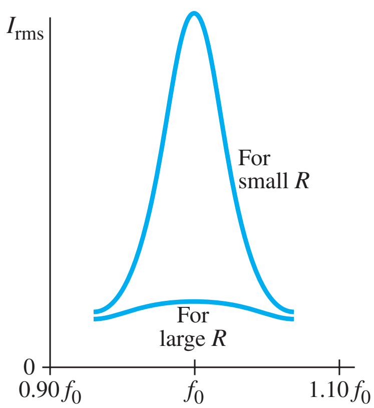

FIGURE 21–46 Current in an LRC circuit as a function of frequency, showing resonance peak at $f = f_0 = 1/(2\pi\sqrt{LC})$.

Because the reactance of inductors and capacitors depends on the frequency f of the source, the current in an LRC circuit depends on frequency. From Eq. 21–18 we see that the current will be maximum at a frequency that satisfies

$$2\pi fL - \frac{1}{2\pi fC} = 0.$$

We solve this for f, and call the solution f_0:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}. \qquad \text{[resonance]} \quad \textbf{(21–19)}$$

When $f = f_0$, the circuit is in **resonance**, and f_0 is the **resonant frequency** of the circuit. At this frequency, $X_C = X_L$, so the impedance is purely resistive. A graph of I_{rms} versus f is shown in Fig. 21–46 for particular values of R, L, and C. For smaller R compared to X_L and X_C, the resonance peak will be higher and sharper.

When R is very small, we speak of an ***LC* circuit**. The energy in an LC circuit oscillates, at frequency f_0, between the inductor and the capacitor, with some being dissipated in R (some resistance is unavoidable). This is called an ***LC* oscillation** or an **electromagnetic oscillation**. Not only does the charge oscillate back and forth, but so does the energy, which oscillates between being stored in the electric field of the capacitor and in the magnetic field of the inductor.

Electric resonance is used in many circuits. Radio and TV sets, for example, use resonant circuits for tuning in a station. Many frequencies reach the circuit from the antenna, but a significant current flows only for frequencies at or near the resonant frequency chosen (the station you want). Either L or C is variable so that different stations can be tuned in (more on this in Chapter 22).

Summary

The **magnetic flux** passing through a loop is equal to the product of the area of the loop times the perpendicular component of the magnetic field:

$$\Phi_B = B_\perp A = BA\cos\theta. \qquad \textbf{(21–1)}$$

If the magnetic flux through a coil of wire changes in time, an emf is induced in the coil. The magnitude of the induced emf equals the time rate of change of the magnetic flux through the loop times the number N of loops in the coil:

$$\mathscr{E} = -N\frac{\Delta\Phi_B}{\Delta t}. \qquad \textbf{(21–2b)}$$

This is **Faraday's law of induction**.

The induced emf can produce a current whose magnetic field opposes the original change in flux (**Lenz's law**).

Faraday's law also tells us that a changing magnetic field produces an electric field; and that a straight wire of length ℓ moving with speed v perpendicular to a magnetic field of strength B has an emf induced between its ends equal to

$$\mathscr{E} = B\ell v. \qquad \textbf{(21–3)}$$

An electric **generator** changes mechanical energy into electric energy. Its operation is based on Faraday's law: a coil of wire is made to rotate uniformly by mechanical means in a magnetic field, and the changing flux through the coil induces a sinusoidal current, which is the output of the generator.

A motor, which operates in the reverse of a generator, acts like a generator in that a **back emf** is induced in its rotating coil. Because this back emf opposes the input voltage, it can act to limit the current in a motor coil. Similarly, a generator acts somewhat like a motor in that a **counter torque** acts on its rotating coil.

A **transformer**, which is a device to change the magnitude of an ac voltage, consists of a primary coil and a secondary coil. The changing flux due to an ac voltage in the primary coil induces an ac voltage in the secondary coil. In a 100% efficient transformer, the ratio of output to input voltages (V_S/V_P) equals the ratio of the number of turns N_S in the secondary to the number N_P in the primary:

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}. \qquad \textbf{(21–6)}$$

The ratio of secondary to primary current is in the inverse ratio of turns:

$$\frac{I_S}{I_P} = \frac{N_P}{N_S}. \qquad \textbf{(21–7)}$$

[*Read/write heads for computer hard drives and tape, as well as microphones, ground fault circuit interrupters, and seismographs, are all applications of electromagnetic induction.]

[*A changing current in a coil of wire will produce a changing magnetic field that induces an emf in a second coil placed nearby. The **mutual inductance**, M, is defined by

$$\mathscr{E}_2 = -M\frac{\Delta I_1}{\Delta t}. \quad \text{(21–8)]}$$

[*Within a single coil, the changing B due to a changing current induces an opposing emf, $\mathscr{E}$, so a coil has a **self-inductance** L defined by

$$\mathscr{E} = -L\frac{\Delta I}{\Delta t}. \quad \text{(21–9)]}$$

[*The energy stored in an inductance L carrying current I is given by $U = \frac{1}{2}LI^2$. This energy can be thought of as being stored in the magnetic field of the inductor. The energy density u in any magnetic field B is given by

$$u = \frac{1}{2}\frac{B^2}{\mu_0}. \quad \text{(21–10)]}$$

[*When an inductance L and resistor R are connected in series to a source of emf, V_0, the current rises as

$$I = \frac{V_0}{R}\left(1 - e^{-t/\tau}\right),$$

where $\tau = L/R$ is the **time constant**. If the battery is suddenly switched out of the LR circuit, the current drops exponentially, $I = I_{\max}e^{-t/\tau}$.]

[*Inductive and capacitive **reactance**, X, defined as for resistors, is the proportionality constant between voltage and current (either the rms or peak values). Across an inductor,

$$V = IX_L, \quad \text{(21–11a)}$$

and across a capacitor,

$$V = IX_C. \quad \text{(21–12a)}$$

The reactance of an inductor increases with frequency f,

$$X_L = 2\pi fL, \quad \text{(21–11b)}$$

whereas the reactance of a capacitor decreases with frequency f,

$$X_C = \frac{1}{2\pi fC}. \quad \text{(21–12b)}$$

The current through a resistor is always in phase with the voltage across it, but in an inductor the current lags the voltage by 90°, and in a capacitor the current leads the voltage by 90°.]

[*In an LRC series circuit, the total **impedance** Z is defined by the equivalent of $V = IR$ for resistance, namely,

$$V_0 = I_0 Z \quad \text{or} \quad V_{\text{rms}} = I_{\text{rms}} Z; \quad \text{(21–14)}$$

Z is given by

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}. \quad \text{(21–15)]}$$

[*An LRC series circuit **resonates** at a frequency given by

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}. \quad \text{(21–19)}$$

The rms current in the circuit is largest when the applied voltage has a frequency equal to f_0.]

Questions

1. What would be the advantage, in Faraday's experiments (Fig. 21–1), of using coils with many turns?
2. What is the difference between magnetic flux and magnetic field?
3. Suppose you are holding a circular ring of wire in front of you and (*a*) suddenly thrust a magnet, south pole first, away from you toward the center of the circle. Is a current induced in the wire? (*b*) Is a current induced when the magnet is held steady within the ring? (*c*) Is a current induced when you withdraw the magnet? For each yes answer, specify the direction. Explain your answers.
4. (*a*) A wire loop is pulled away from a current-carrying wire (Fig. 21–47). What is the direction of the induced current in the loop: clockwise or counterclockwise? (*b*) What if the wire loop stays fixed as the current I decreases? Explain your answers.

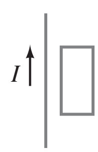

FIGURE 21–47 Question 4.

5. (*a*) If the north pole of a thin flat magnet moves on a table toward a loop also on the table (Fig. 21–48), in what direction is the induced current in the loop? Assume the magnet is the same thickness as the wire. (*b*) What if the magnet is four times thicker than the wire loop? Explain your answers.

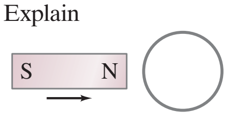

FIGURE 21–48 Question 5.

6. Suppose you are looking along a line through the centers of two circular (but separate) wire loops, one behind the other. A battery is suddenly connected to the front loop, establishing a clockwise current. (*a*) Will a current be induced in the second loop? (*b*) If so, when does this current start? (*c*) When does it stop? (*d*) In what direction is this current? (*e*) Is there a force between the two loops? (*f*) If so, in what direction?
7. The battery mentioned in Question 6 is disconnected. Will a current be induced in the second loop? If so, when does it start and stop? In what direction is this current?
8. In Fig. 21–49, determine the direction of the induced current in resistor R_A (*a*) when coil B is moved toward coil A, (*b*) when coil B is moved away from A, (*c*) when the resistance R_B is increased but the coils remain fixed. Explain your answers.

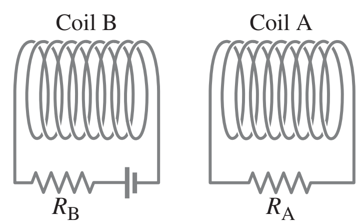

FIGURE 21–49 Question 8.

9. In situations where a small signal must travel over a distance, a *shielded cable* is used in which the signal wire is surrounded by an insulator and then enclosed by a cylindrical conductor (shield) carrying the return current. Why is a "shield" necessary?
10. What is the advantage of placing the two insulated electric wires carrying ac close together or even twisted about each other?
11. Explain why, exactly, the lights may dim briefly when a refrigerator motor starts up. When an electric heater is turned on, the lights may stay dimmed as long as the heater is on. Explain the difference.
12. Use Figs. 21–14 and 21–17 plus the right-hand rules to show why the counter torque in a generator *opposes* the motion.
13. Will an eddy current brake (Fig. 21–20) work on a copper or aluminum wheel, or must the wheel be ferromagnetic? Explain.